Cairo University
Faculty of Engineering
Elec. & Comm. Dept.

Linear Systems Theory
3rd Year, 2011 / 2012

<u>Sheet (9): Controlability, Observability, and State Feedback</u>

(1) Given the following state space model

$$\underline{\dot{X}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -21 & -16 & -6 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U \ ; \quad y = \begin{bmatrix} 10 & 15 & 5 \end{bmatrix} \underline{X}$$

Test this model for both controllability and observability.

(2) A system is described by:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Test the system for both controllability and observability.

(3) Consider the system characterized by the D.E:

$$\ddot{y}(t) - \dot{y}(t) - 6y(t) = \dot{u}(t) - 3u(t)$$

By choosing the states:

x_1=y , $x_2 = \dot{y} - y - u$

a- Write down the state equations.
b- Test the stability of the system
c- Check controllability and observability of the system.

(4) Given a system described by

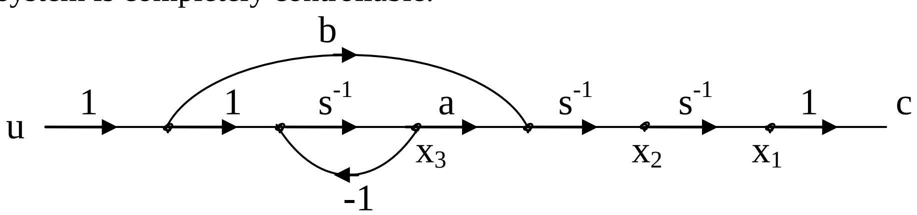
$$\underline{\dot{X}} = \begin{bmatrix} -1 & 3 \\ 4 & -2 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \underline{X}$$

Test the system stability, Controllability and observability.

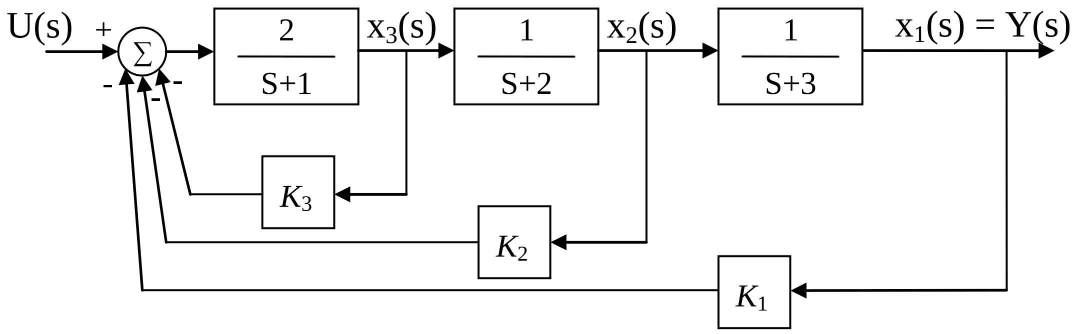
(5) *(Jan2001)* the state diagram of a linear control system is shown in the following figure

- For a=1 ,b=2 , determine the state controllability and observability of the system.
- Determine the relation between the parameters a and B that should be avoided in order that the system is completely controllable.



(6) *(Dec2002)* an automobile suspension system has three physical state variables as shown in the following figure.

- Develop the state space representation of the system.
- Test the controllability and observability of the obtained state model.
- Select K_1,K_2 and K_3 so that the roots of the characteristic equation are placed at: -2,-2,-6.



(7)*(Jan2002)* Consider the system

$\dot{\underline{X}} = A\underline{X} + B$, y= C $\underline{X}$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ C = [1 \ \ 0 \ \ 0]$$

Check the controllability and observability of the system.

By using state feedback control $u(t) = -\underline{k}\,\underline{X}(t)$, where k is state feedback vector, it is desired to have the closed loop poles at:

λ_1= -2+j3.464, λ_2 = -2-j3.464, λ_3 = -5

Determine the state feedback gain vector and the control signal