Cairo University
Faculty of Engineering
Elec. & Comm. Dept.

Linear Systems Theory
3rd Year, 2011 / 2012

Sheet (8): Stability

(1) Test the stability of the systems described by the following differential equations:

(a) $\dddot{y} + 3\ddot{y} + \dot{y} - 2y = u(t)$

(b) $y^{(4)} + 2\dddot{y} + 5\ddot{y} + \dot{y} + 2y = u(t)$

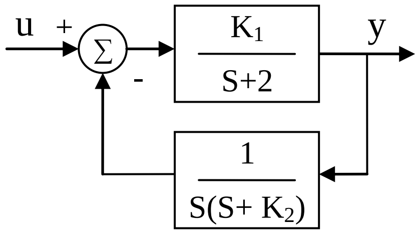
(c) $\dddot{y} + \alpha\ \ddot{y} + \beta\,\dot{y} + \delta\, y = u(t)$

(2) Show that a system with the characteristic equation:

$$(\alpha\text{-}2)\, s^3 + (1-\alpha)\, s^2 + (\alpha+5)\, s + (\alpha-3) = 0$$

is always unstable for any value of α.

(3) For the system shown in figure (1), find the conditions on k_1 and k_2 to make the system stable. Plot the region of stability for K_1 and K_2.



(4) Given the closed-loop system described by the state equations:

$$\underline{\dot{X}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2k & -2 & -3 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} U$$

(i) Using Routh criterion, find the range of K for which the system is stable.

(ii) Determine the value of K that will result in a marginally stable system, find the frequency of oscillation.

(iii) Find the location of all roots of the system characteristic equation for that value of K found in (ii).

(5) (*midterm2003*) The design of a turning control for a tracked vehicle involves the selection of two parameters. The block diagram of the system model is shown in figure 2 where R(s) is the desired direction of turning and D(s) is the disturbance.

- find the condition on ***a*** and ***k*** for stable operation. Plot the region of stability on ***k-a*** plane.
- If the value of ***a*** is selected to be (***a*=1**), select a gain ***K*** that will result in a stable operation. find the value of ***K*** that gives critically stable system and the frequency of oscillation.