Cairo University
Faculty of Engineering
Elec. & Comm. Dept.

Linear Systems Theory
3^{rd} Year, 2011 / 2012

<u>Sheet (7): System Solution in SSR</u>

(1) A system is characterized by the state equation

$$\dot{\underline{x}} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}\underline{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

Where u(t) = unit step input. Compute the solution of the state vector if the initial conditions are $X_1(0) = 1$ and $X_2(0) = -1$.

(2) Compute the solution of state vector for the system

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 2 \\ -3 & -5 \end{bmatrix}\underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

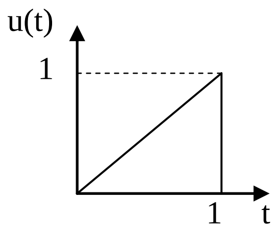
Given that $X(0)^T = [\,3 \quad 1\,]$ and $u(t) = e^{-t}$.

(3) For the system described by

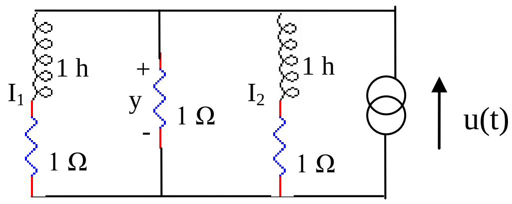
$$\dddot{y} + 3\ddot{y} + 2\dot{y} = u$$

Derive two state space representations, then solve the system for the initial conditions y(0) = 0, $\dot{y}(0) = 0$, $\ddot{y}(0) = -1$ and u = unit step input.

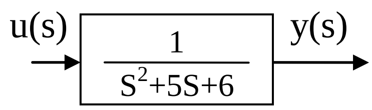
(4) For problem (6) sheet (5) get the response of the system for the given initial conditions and the input u(t) is as shown.



(5) Write the state and output equations for the circuit shown.
Let $\underline{X}^T = [i_1 \quad i_2]$ and find:
(a) The impulse response.
(b) The step response.



(6) Consider the system shown. It is required to



(a) Find the SSR of the system, using the following state variables: $X_1 = y$, $X_2 = \dot{y}$.
(b) Find the state transition matrix ϕ(t) associated with the representation of part (a).
(c) Verify that $\dot{\phi}(t) = A\phi(t)$.
(d) Find the free (no input) response for $X_1(0) = 0$ and $X_2(0) = 1$.
(e) Find the system's response for the same initial conditions as in part (d) with u(t)=1.
(f) Find the SSR such that the matrix A is diagonal.
(g) Verify the results of parts (d) and (e) using the representation of (f).