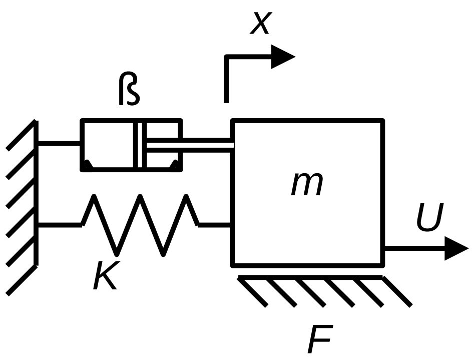
Cairo University
Faculty of Engineering
Elec. & Comm. Dept.

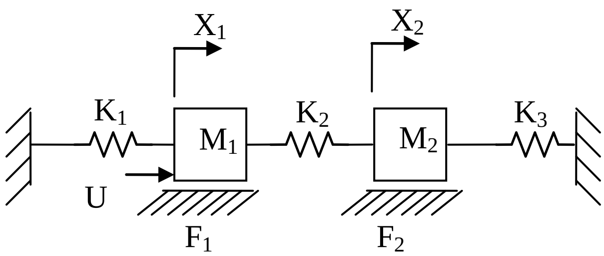
Linear Systems Theory
3^{rd} Year, 2011 / 2012

Sheet(2): Modeling

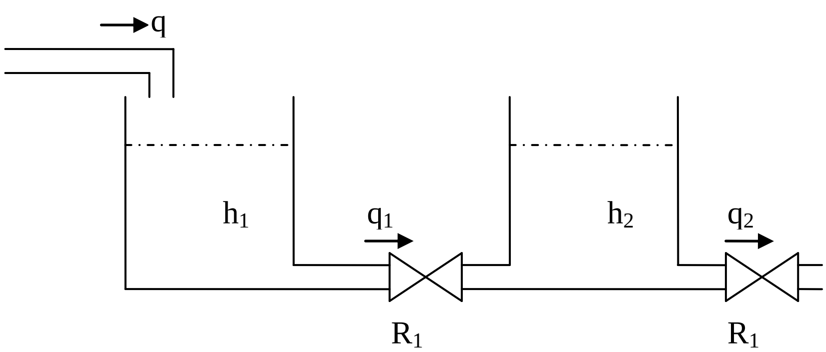
1. Derive the differential equations that represent the mechanical system shown, where U is a force applied on the mass m. Hence, derive the transfer function $x(s)/U(s)$.



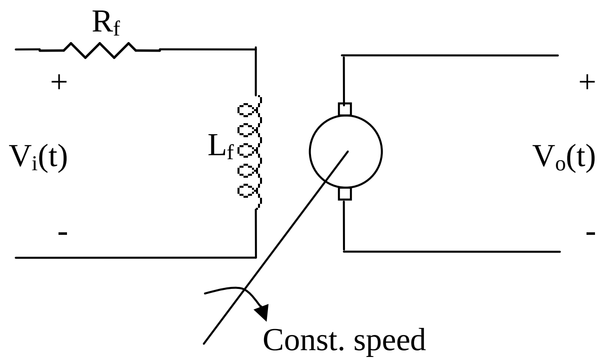
2. Derive the differential equations that represent the mechanical system shown, where U is a force that affects the mass M_1 and hence derive the transfer functions $X_1(s)/U(s)$ and $X_2(s)/U(s)$. If the force U has a value of 1 N find the steady state values of X_1 and X_2.



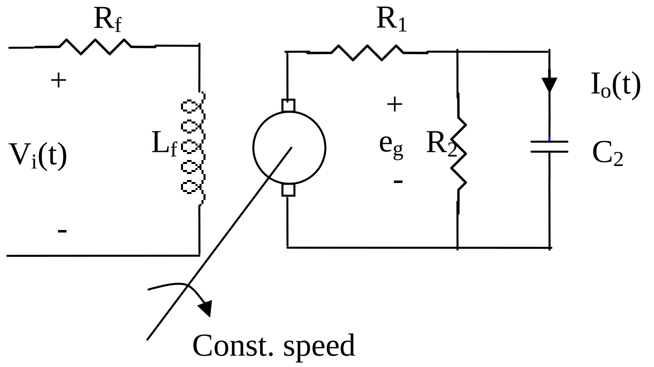
3. The figure shows a process plant containing of two tanks of areas A_1 and A_2, and having steady state flow rate of Q. suppose that the incoming flow rate changes to Q+q, and that the corresponding changes in the flow rate to the second tank and at the outlet become $Q+q_1$ and $Q+q_2$, respectively. Derive the transfer function that relates q_2 to q.



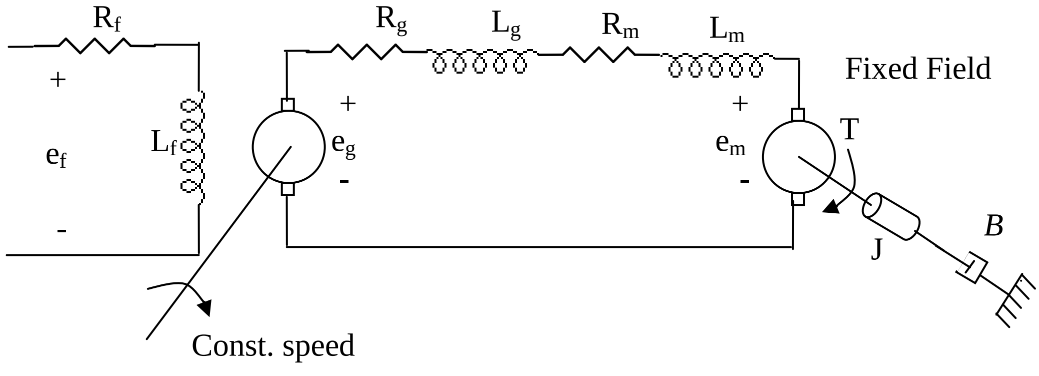
4. The shown figure shows a DC shunt wound generator, rotating at a constant speed, with a voltage signal applied to its field winding. Drive a mathematical model for determining the generator output voltage. Then, find the T.F = Vo(s)/Vi(s).



5. The system shown in the figure consists of a DC- generator connected to RC impedance. Find the differential equation and the T.F that relates V_i and I_o.



6. A schematic diagram of a generator-motor system driving an inertia load *J* with viscous damping *B* at an angular velocity θ is shown in the figure. Given that the torque constant of the motor and its induced back emf constants are K_T and K_m respectively and the generator constant is K_g. Derive a mathematical model for determining the motor output speed and hence find the transfer function of the system.



7. The figure shows a level control process. We wish to ensure that the level h in the vessel follows a desired value h_r even when there is variations in the outlet flow rate q_o . The different components have the following characteristics:

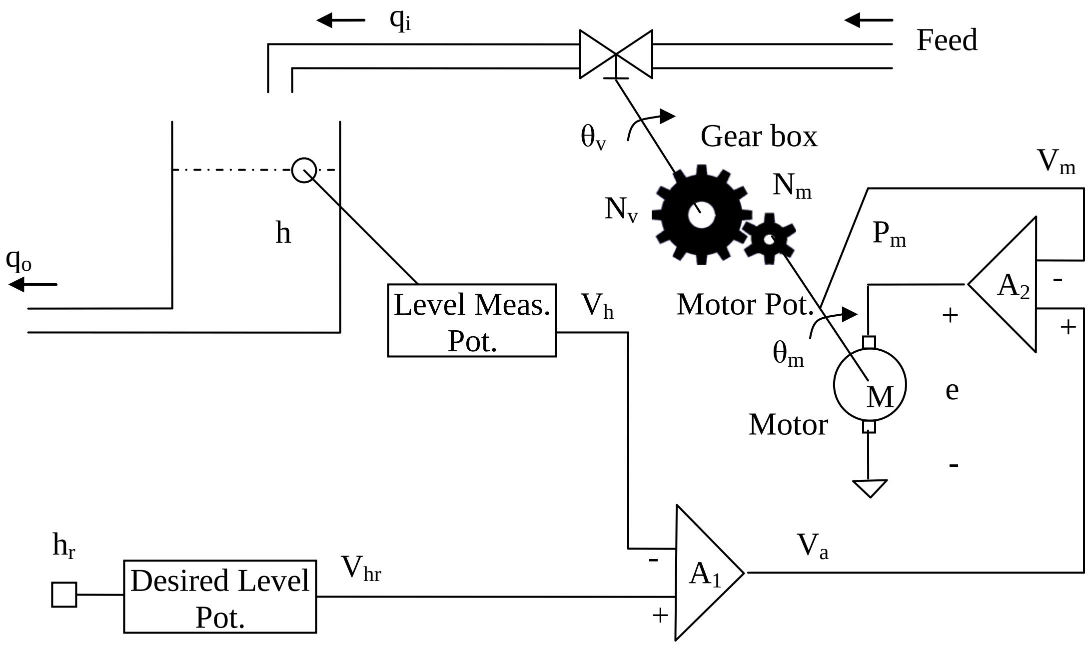
- Vessel: Max level h_{max} =0.5 m, area of section = 0.25 m^2.
- Level measuring instrument: Output voltage is proportional to level with a constant λ, where λ = 20 volts/m.
- Desired level potentiometer: Graduated from 0 to h_{max} , its output voltage is the same as above.
- Amplifiers: Gains A_1 & A_2 adjustable. Their B.W and i/p impedances are infinite, o/p impedances=0.
- Motor: The T.F of the motor system can be approximated by

$$\frac{\theta_m(s)}{e(s)} = \frac{K_m}{s(1+\tau_m s)},$$

where K_m= 0.5 radian/sec/volt and τ_m = 0.1 sec.
- Motor potentiometer: It provides a voltage proportional to the angular position of the motor shaft θ_m with a constant K_p= 1 volt / radian.
- Gearbox: $N_v/N_m = 20$.
- Valve: The flow rate q_i is proportional to the angular position θ_v with a flow rate coefficient K_v = 0.1 m^3/sec/rad.

Assuming that the outlet flow q_o is independent of the level h in the vessel, answer the following:

a) Show how the system operates.
b) Draw a functional block-diagram for the whole system. Identify components and variables.
c) Deduce the T.F θ_m/V_a , and the T.F of the closed-loop system h/h_r.



8. The figure shows a position control system. Assume that the input to the system is the reference shaft position θ_R and the system output is the load shaft position θ_c, the parameters of the system are given below:

–Draw a complete block diagram for the system.
–Obtain the overall transfer function θ_C/θ_R.

<u>Assume the following data:</u>
I/P potentiometer const. = o/p potentiometer const. = K_p = 10 volt/rad.
DC Amplifier gain = K_A =50.
Motor field resistance = R_f = 100Ω.
Motor field inductance = L_f = 20 Henry.
Motor torque const. = K_T = 10 Newton meter / Ampere (Nm/Amp.)
Moment of inertia of the load = J_L =250 Kg.m^2.
Coefficient of viscous Friction of the load = B_L = 2500 Nm per rad./sec.
Motor to Load gear ratio:

$$\frac{\theta_L}{\theta_M} = \frac{N_M}{N_L} = \frac{1}{50}$$

Load to potentiometer gear ratio:

$$\frac{\theta_C}{\theta_L} = 1$$