Cairo University
Faculty of Engineering
Elec. & Comm. Dept.

***Linear Systems Theory***
3rd Year, 2011 / 2012

# Sheet(1): Revision

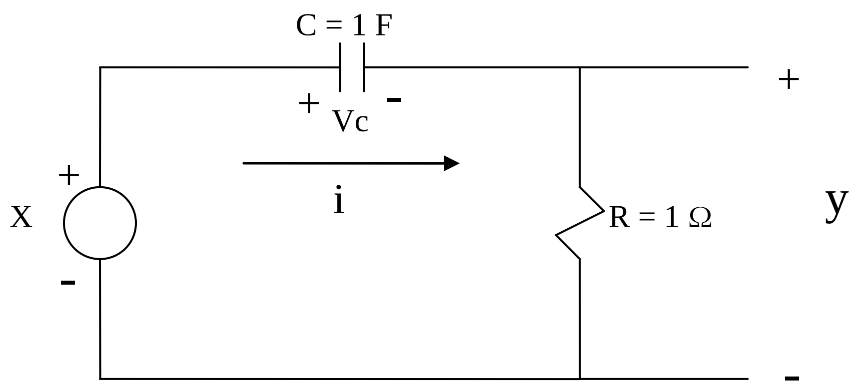
**1. Solve the following differential equations:**

a) $\ddot{y}(t)+3\dot{y}(t)+2y(t)=u(t)=unit\ step$, assume the initial conditions: $y(0)=-1, \dot{y}(0)=2$

b) $\ddot{y}(t)+3\dot{y}(t)+2y(t)=\dot{x}(t)+3x(t)$, assume the initial conditions: $y(0)=1$, $\dot{y}(0)=0$ and the input is given by: $x(t)=e^{-4t}$

c) $\dot{x}(t)=a\,x(t)+b\,u(t)$, where a,b are constants, u(t) is an arbitrary time function and x(0) is the initial value of x(t) at t = 0.

**2. For the RC network in the schematic given below:**

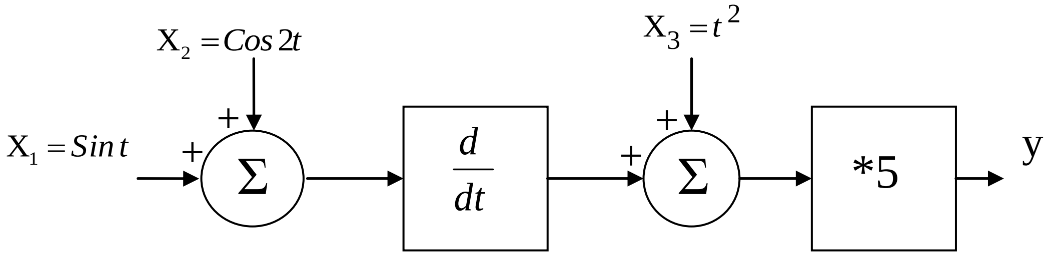



a) Find a differential equation that relates the output voltage 'y' and the input voltage 'x'.
b) Let the initial voltage across the capacitor C be Vc = 1volt with the polarity shown, and let $x(t)=2e^{-t}$. Using the Laplace Transform technique, find 'y'.

**3. Test the linearity of the systems described by the following i/p – o/p relations:**

a) $y(t)=au(t)$, where 'a' is a constant.
b) $y(t)=u^3(t)$
c) $y(t)=e^{u(t)}$
d) $\dot{x}(t)=a\,x(t)+b\,u(t)$, x(0) = 0, a,b are constants
e) $\ddot{y}(t)+a\,\dot{y}(t)+y(t)=u(t)$, $y(0)=\dot{y}(0)=0$

**4. Determine the output 'y' of the following system:**



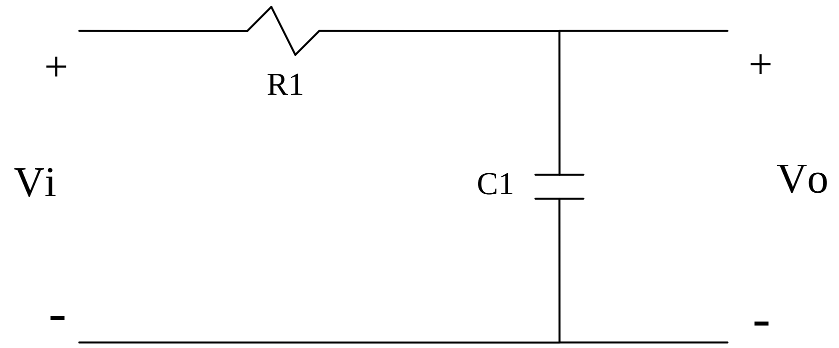

**5. Find the Transfer Function of the following systems:**

a) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \dot{x}(t) + 3x(t)$

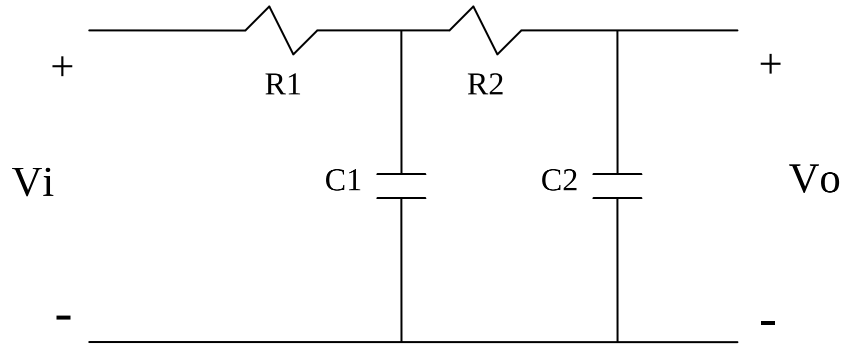
b) $\dot{y}(t) + y(t) = x(t-T)$

**6. Derive the Transfer Function of the following RC circuits:**
(hint: Solve directly in the complex domain (Capacitance → 1/SC) and assume zero initial conditions)

**a)**




**b)**



# Summary

1. Properties of linear systems
2. Laplace transform

   For a time function f(t) ,Laplace transformation is obtained by evaluating

$$F(s)=\int_{-\infty}^{\infty} f(t)e^{-st}dt$$

3. Famous Laplace Transform Pairs

| Time Function | Laplace transform |
|---|---|
| Unit step u(t) | $\frac{1}{S}$ |
| Ramp function tu(t) | $\frac{1}{S^2}$ |
| $t^n$ | $\frac{n!}{S^{n+1}}$ |
| $e^{-\alpha t}$ | $\frac{1}{S+\alpha}$ |
| $e^{-\alpha t}f(t)$ | $F(s+\alpha)$ |
| $\cos(at)$ | $\frac{S}{S^2+a^2}$ |
| $f(t-t_0)$ | $e^{-st_0}F(s)$ |
| $\dot{f}(t)$ | $SF(S)-f(0)$ |
| $\ddot{f}(t)$ | $S^2F(S)-Sf(0)-\dot{f}(0)$ |
| $\int f(t)dt$ | $\frac{F(S)}{S}$ |
| $-tf(t)$ | $\frac{d}{dS}F(s)$ |
| Convolution $f_1(t)*f_2(t)$ | Multiplication $F_1(s)*F_2(s)$ |

4. Initial and Final value theorems

   Initial value theorem

$$f(0)=\lim_{S\to\infty} SX(s)$$

   Final value theorem

$$f(\infty)=\lim_{S\to 0} SX(s)$$

5. Transfer Function

   Transfer function is a relation between system output Y and system input X in S-domain

$$TF=\frac{Y(s)}{X(s)}$$