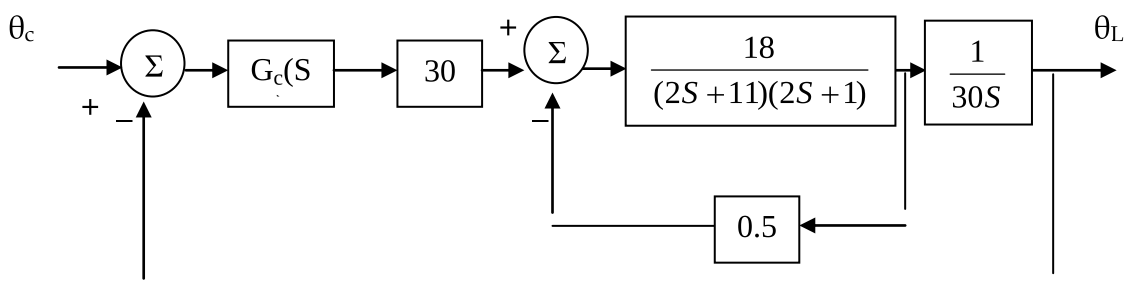
SHEET 9

Dynamic Compensation using root locus method

[1] (Final2001)For the system shown in the figure below:

a. Sketch the root locus of the uncompensated system.(With the compensator replaced by a gain K).

b. Using the root locus plot, design a compensator to give a characteristic equation root at S = -1 + j.



[2] (Final2003)The tilt control system has unity feedback and a forward transfer function:

$$G(S) = \frac{20K}{(S+1)(S^2+8S+20)}$$

a. Sketch the root locus of the system as K varies from 0 to ∞.

b. From the plot determine the value of K that gives the complex roots a damped natural frequency equal 3 rad/sec. Predict the response of this system to a step input.

c. Design a series controller that gives the system dominant complex roots at: S = -3 ± j3

[3] A servomechanism position control has the plant transfer function:

$$G(S)=\frac{10}{S(S+1)(S+10)}$$

You are to design a series compensation transfer function $G_c(S)$ in the unity feedback configuration to meet the following closed loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.
- The response to a reference step input is to have a rise time of no more than 0.4 sec.

[4] A model for a space vehicle control system has the following open loop transfer function: $G(s)H(s)=\frac{1}{s^2(0.1s+1)}$.

Design a series compensator such that:

- The damping ratio ζ =0.5
- The undamped natural frequency ω_n = 2 rad\sec.

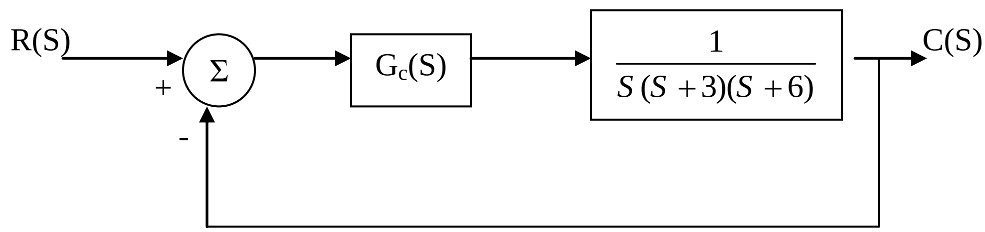
[5] A certain plant with a transfer function: $G(s)=\frac{16}{s(s+4)}$ is in a unity feedback system.

Design a compensator such that the static velocity error constant is 20 sec^{-1} without changing the original location of the closed loop poles.

[6] A certain plant with a transfer function: $G(s) = \frac{1}{s(s+2)}$ is in a unity feedback system.

Design a lag compensator so that the dominant poles of the closed loop system are located at S= -1 ± j and the steady state error to a unit ramp input is less than 0.2.

[7] According to the below figure :



Design a lag Compensator $G_c(S)$ to meet the following specifications:

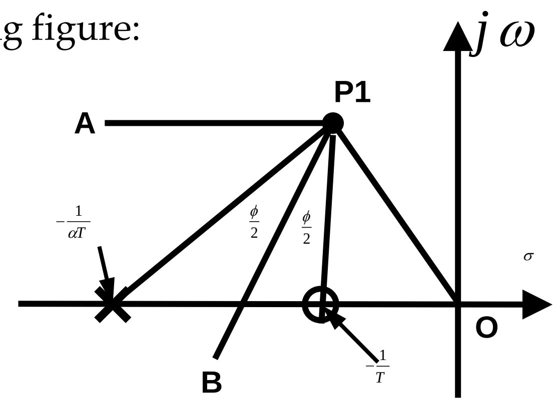
- The step response settling time is to be less than 5 sec.
- The step response overshoot is to be less than 17%.
- The steady state error to unit ramp input must not exceed 10%.

[1] Lead compensation design procedure

1. Determine the desired location for the dominant closed-loop poles (P_1 and P_2).
2. Plot the root-locus of the uncompensated system.
3. Check whether or not the gain adjustment alone can yield the desired closed loop poles.
4. If not, determine the angle deficiency ϕ:

$$\phi = -180 - \angle G(S)\Big|_{\text{the desired closed loop pole =P1}}$$

5. If $\phi > 55^0$ → you must use double lead compensator.
6. from the following figure:



Determine T and α for the compensator:

$$G_C(s) = K_C \frac{s + 1/T}{s + 1/(\alpha T)} \qquad 0 < \alpha < 1$$

7. Determine the open-loop gain of the compensated system from the magnitude condition as:

$$\left| G_c(S) G(S) \right|_{\text{at the desired closed loop pole=P1}} = 1$$

$$then, \left| K_c \frac{S + 1/T}{S + 1/\alpha T} G(S) \right|_{\text{at the desired closed loop pole=P1}} = 1$$

[2] Lag compensation design procedure:

1. Determine the desired location for the dominant closed-loop poles from transient response specs, which will specify the value of K used in the root locus before compensation.
2. Assume the transfer function of the lag compensator to be:

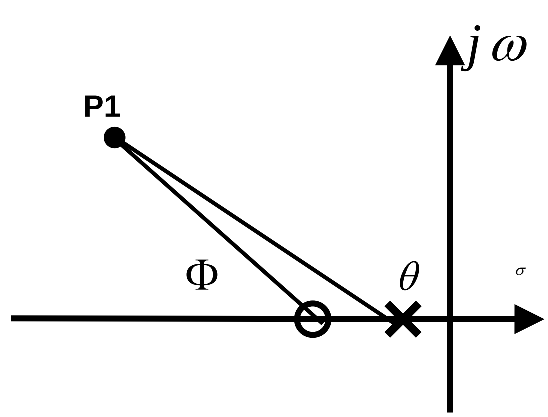
$$G_C(s) = \hat{K}_C \frac{s + 1/T}{s + 1/(\beta T)}$$

3. Evaluate the static error constant as required in the problem.
4. Determine β as :

$$\beta = \frac{\textit{static}\ \text{error constant}_{\text{new}}}{\textit{static}\ \text{error constatnt}_{\text{old}}}$$

5. Determine the pole and zero of the lag compensator using the following criteria:

"Place the pole and the zero very near to each other and very close to the origin, such that the angle contribution of the lag system is very small→ $\Phi - \theta \approx (5^\circ - 7^\circ)$"



6. Adjust the gain of the compensator for magnitude condition