SHEET 8

Nyquist Diagram

[1] A unity feedback system has a forward transfer function: $G(s)=\dfrac{k}{s(s+1)(s+0.5)}$

a) Sketch the Nyquist diagram for k=1, 0.75 and 0.5.
b) For the above cases, determine :
i)Whether the system is stable. Check using Routh method.
ii) The gain crossover and the phase crossover frequencies.
iii)The gain and phase margins.
c) Adjust the gain such that the GM=2db.
d) Adjust the gain such that the PM=10 degrees.

[2] A unity feedback system has a forward transfer function: $G(s)=\dfrac{(s+2)}{s^2(s+0.5)}$

a) Sketch the Nyquist diagram.
b) For the above cases, determine :
i)Whether the system is stable. Check using Routh method.
ii) The gain crossover and the phase crossover frequencies.
iii)The gain and phase margins.
c) Adjust the gain such that the GM=2db.
d) Adjust the gain such that the PM=10 degrees.

[3] (midterm2001/Final 2002) A system with unity feedback has the forward transfer function: $G(S)=\dfrac{K}{(S+1)(S^2+2S+2)}$

(a) Find the limiting value of K for stability.
(b) Sketch the Nyquist plot for K = 2.5 based on points calculated at ω = 0, 0.5, 1, 1.5, 1.8, 2 and ∞ rad/sec.
(c) For the value of K obtained in (b), find the gain crossover frequency ω_{gc} , the phase crossover frequency ω_{pc}, the gain margin GM, the phase margin PM and steady-state error e_{ss} for a unit step input.
(d) Determine the new gain corresponding to a phase margin of 30^o, and the new steady-state error $e_{ss\ new}$ for a unit step input.

[4](Midtem97/2002) In a frequency response experiment, it was found that:
Gain crossover frequency, w_{gc}=1.2 rad/sec and Phase Margin=22.92°.

(a) Assume that the closed loop system is dominated by a second-order model, find the closed loop transfer function of this system. (Note : Prove any formula used)
(b) Assume that the above system is unity feedback and insert $G^*(s)=k/(s+1)$ in the forward path. For a desired gain margin of 20 db.
Determine:

- The static error coefficient k.
- The corresponding steady state error for a unit ramp input.
- The new phase margin.

[5](Final2001)
(a) The following experimental results were obtained from an open loop frequency response of an automatic control system.

ω (rad/sec)	4	5	6	8	10
Gain	0.66	0.48	0.36	0.23	0.15
Phase (°)	-134	-143	-152	-167	-180

(i) Plot the Nyquist diagram of the open loop T.F.
(ii) Obtain the values of ω_{pc}, GM and PM.(You can extrapolate the Nyquist locus).

[6] (Final2000) For a unity feedback type 1 control system, the forward transfer function is given by $G_p(S) = \dfrac{K}{S(1+S)(1+0.1S)}$. Using Nyquist plot,
(a) Obtain the values of gain crossover frequency ω_{gc}, phase crossover frequency ω_{pc}, gain margin GM and phase margin PM for the normalized plant.
(b) Find the limiting value of K for the above closed loop system to be marginally stable. Verify your result using Routh's stability criterion.
(c) Estimate the corresponding values of ζ, ω_n and the bandwidth.
(Prove any formula you use)

Summary

I] Main steps:

1. If you have a transfer function in the form of: $G(S)=\dfrac{K(S+z_1)(S+z_2).....(S+z_m)}{S^r(S+p_1)(S+p_2).....(S+p_n)}$

2. Rearrange the transfer function in the form of:
$G(S)=\dfrac{K(1+j\,\omega\tau_a)(1+j\,\omega\tau_b).....(1+j\,\omega\tau_m)}{S^r(1+j\,\omega\tau_1)(1+j\,\omega\tau_1).....(1+j\,\omega\tau_n)}$

3. determine the type of the system:

   a. *<u>For type 0 system(r = 0):</u>* The starting point of the plot is finite and is on the positive real axis and equals K.

   b. *<u>For type 1 system(r = 1):</u>* The starting point of the plot is infinite and is asymptotic to a line parallel to the negative imaginary axis.

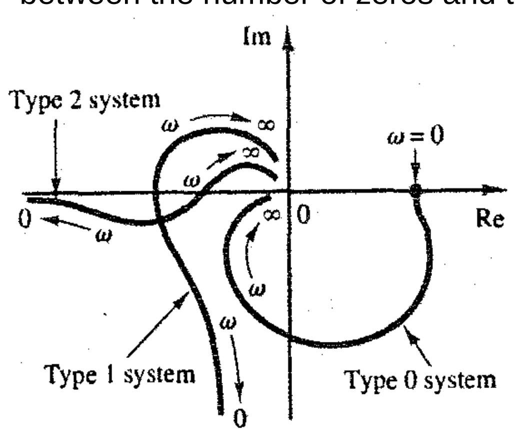
   c. *<u>For type 2 system(r = 2):</u>* The starting point of the plot is infinite and is asymptotic to a line parallel to the negative real axis.

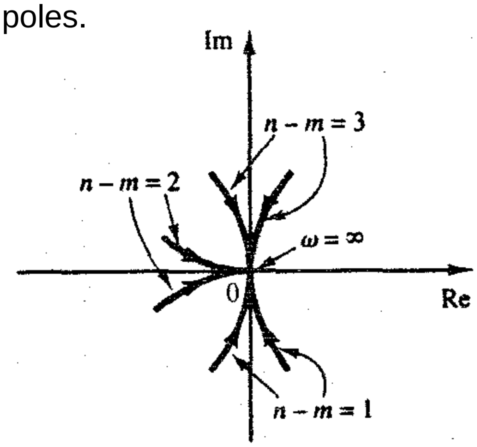
4. get the magnitude and the phase of the transfer function in the form:

$$|G(S)|=\frac{K\sqrt{1+(\omega\tau_a)^2}\sqrt{1+(\omega\tau_b)^2}...}{\omega^r\sqrt{1+(\omega\tau_1)^2}\sqrt{1+(\omega\tau_2)^2}...}$$

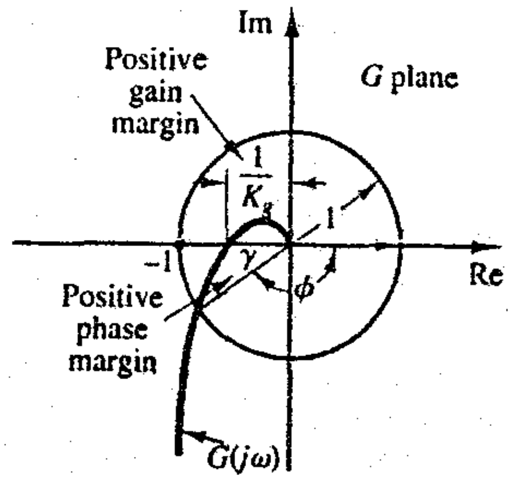
$$\underline{|G(S)} = -90r+\tan^{-1}\omega\tau_a+\tan^{-1}\omega\tau_b+....$$
$$-\tan^{-1}\omega\tau_1-\tan^{-1}\omega\tau_2-....$$

5. substitute in the $|G(S)|\, and\, \underline{|G(S)}$ by:
   a. All corner frequencies.
   b. $\omega=0$ and $\omega=\infty$.
   c. You must have **3 points** at least in the third quarter and at least **one point** in the second quarter (if the plot exists in these regions).
6. At $\omega=\infty$ the plot will be tangent to one of the axes, according to the difference between the number of zeros and the number of poles.





II] Stability analysis using Nyquist diagram:



- GM = $\frac{1}{K_g}$ or GM_{db} = -20 log K_g .
- PM = 180 + $\left.\underline{|G(S)}\right|_{\omega gc}$ =180 + Φ
- How to get ω_{gc} ?
    1. $\left.\underline{|G(S)}\right|_{\omega gc}$ = Φ
    2. Take tan for the two sides.

    (Remember $\tan(a \pm b) = \frac{\tan a + \tan b}{1 \mp \tan a \tan b}$)
    3. Solve to get ω_{gc} .
- How to get ω_{gc} ?
    1. $\left.\underline{|G(S)}\right|_{\omega pc}$ = -180
    2. Take tan for the two sides.(Remember $\tan(a \pm b) = \frac{\tan a + \tan b}{1 \mp \tan a \tan b}$)
    3. Solve to get ω_{pc} .

III] System identification:

1. For a second order:

- If you are given the frequency response (ω_{gc} & PM)→ and required the time response (ζ & ω_n).
- $\frac{\omega_{gc}}{\omega_n} = \sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}$
- $PM = \tan^{-1} \frac{2\zeta\omega_n}{\omega_{gc}}$
- $\tan PM = \frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}}$
- For PM $\leq$ 60^o → $\zeta \approx \frac{(PM)}{100}$.
- M_m or $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$
- ω_m or $\omega_r = \omega_n\sqrt{1-\zeta^2}$
- $\omega_B = \sqrt{\omega_n{}^2(1-\zeta^2) + \omega_n\sqrt{2(1-\zeta^2\omega_n{}^2)}}$