Cairo University
Faculty of Engineering
Elect. & Comm. Depart.

Control Exam
Third Year
Time: 1:1/4 Hour

الرقم:	الفصل:	الاسم(باللغة العربية):

(1) A system to aid and control the walk of a partially disabled person cold use automatic control of the walking system. One model of the system is shown in figure 2. Design a controller $G_c(s)$ such that the settling time to a unit step reference input is approximately 4 second.

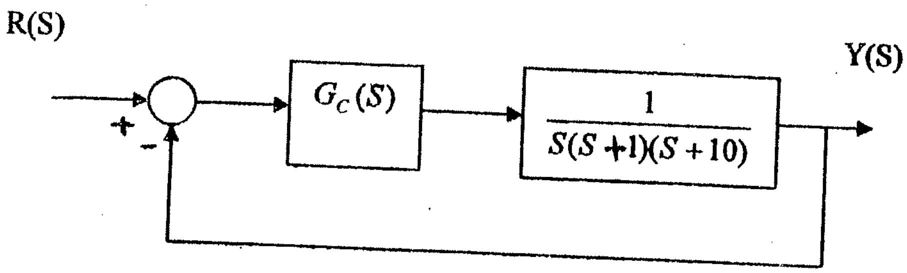


Figure 2

-Find the improvement in the steady state error due to a unit step reference input.
-Sketch the resulting time response to a unit step input.
-Comment on the effect of the controller $G_c(s)$ on the time response.
-If the system is subjected to a unit step disturbance input between the plant and the controller, find the improvement in the steady state error due to the unit step disturbance input.

Midterm Solution

Q1 Solution:

* To acheive the required settling time we will use a PD controller.

$$\boxed{G_c(S) = K_p + K_d S}$$

After adding the compensator, the characteristic equation will be

$$\frac{K_p + K_d S}{S(S+1)(S+10)} + 1 = 0$$

$$S^3 + 11 S^2 + (10 + K_d) S + K_p S = 0 = (S+\alpha)(S^2 + 2\eta\omega_n S + \omega_n^2)$$

to have $t_s = 4\,sec = \frac{4}{\eta\omega_n} \rightarrow \therefore \; \eta\omega_n = 1$ ①

Comparing coefficients:

$\alpha + 2\eta\omega_n = 11 \longrightarrow \alpha = 9$

$\omega_n^2 + 2\eta\omega_n\alpha = 10 + K_d \rightarrow K_d = \omega_n^2 + 8$ ②

$\alpha\omega_n^2 = K_p \rightarrow K_p = 9\omega_n^2$ ③

we have 3 equations in 4 unknowns $(\eta, \omega_n, K_p, K_d)$

* As this application is for the aid of a disabled person the damping factor should be large enough to reduce the oscillations. So we will take $0.7 < \eta < 1$

$\rightarrow$ for $\eta = 0.707 = \frac{1}{\sqrt{2}}$

$\longrightarrow \omega_n = \sqrt{2}$ rad/sec

$\longrightarrow K_d = 10$

$\longrightarrow K_p = 18$

for $\zeta = 1 \longrightarrow \omega_n = 1$ rad/sec

$\longrightarrow K_d = 9$

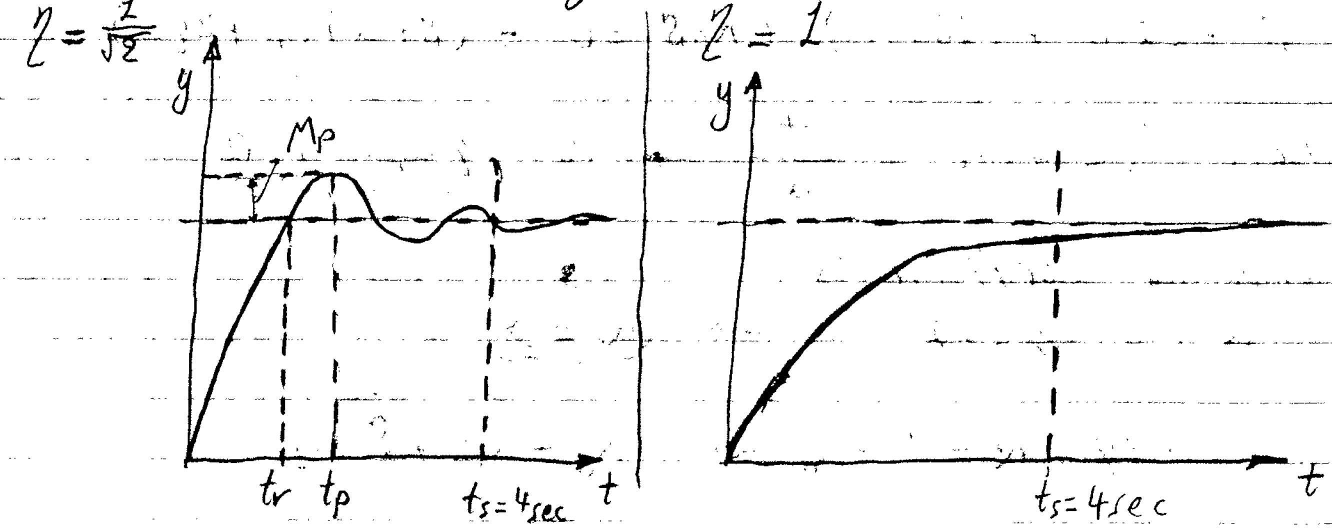
$\longrightarrow K_p = 9$

☆ Improvement in SS error for a Unit step input

→ the system is type 1 before & after the Compensation, so the SS error for a Unit step is Zero in both cases.

NO Improvement !

☆ Sketch the resulting time response



$M_p = 4.3\ \%$

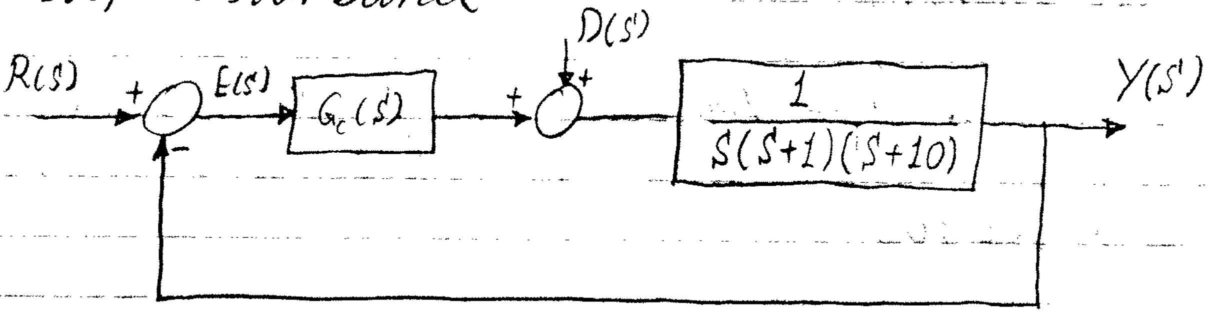
$t_r = \frac{3}{4}\pi = 2.356$ sec

$t_p = \pi = 3.14$ sec

Comment:

The PD controller responds to the rate of change of the error. It adds damping to the system & and improves the transient response. The PD controller has no direct effect on the steady state error.

☆ Unit step disturbance



let $R(S) = 0$

$$\frac{E(S)}{D(S)} = -\frac{\dfrac{1}{S(S+1)(S+10)}}{1 + \dfrac{G_c(S)}{S(S+1)(S+10)}} = \frac{-1}{S^3 + 11S^2 + 10S + G_c(S)}$$

$$e_{ss} = \lim_{S \to 0} S\,E(S) \qquad \text{but } D(S) = \frac{1}{S}$$

$$\therefore\ e_{ss} = \lim_{S \to 0} \frac{E(S)}{D(S)} = -\frac{1}{\lim_{S \to 0} G_c(S)}$$

before Compensation $G_c(S) = 1 \longrightarrow e_{ss} = -1$

after Compensation $G_c(S) = K_p + K_d S \longrightarrow e_{ss} = \frac{-1}{K_p}$

$\zeta = 1 \longrightarrow e_{ss} = \frac{-1}{9}$

$$\text{Improvement} = \frac{e_{ss}|_{before} - e_{ss}|_{after}}{e_{ss}|_{before}}$$

$$= \frac{1 - \frac{1}{9}}{1}$$

89 % improvement in SS error due to unit step disturbance

$\zeta = \frac{1}{\sqrt{2}} \longrightarrow e_{ss} = \frac{-1}{18}$

$$\text{Improvement} = \frac{1 - \frac{1}{18}}{1}$$

94 % improvement in SS error due to unit step disturbance

(2) The open loop transfer function of a unity feedback control system is

$$G(s) = \frac{K}{s(1+0.1s)(1+0.4s)}$$

(a) Use the Bode plot to estimate the gain and phase margins of the system. Discus the closed- loop stability

(b) Design a lag-lead compensator so that the phase margin is at least 45^0, and evaluate the system band-width before and after compensation.

(c) The velocity constant, $K_v = 100\ sec^{-1}$

Q2 Solution :

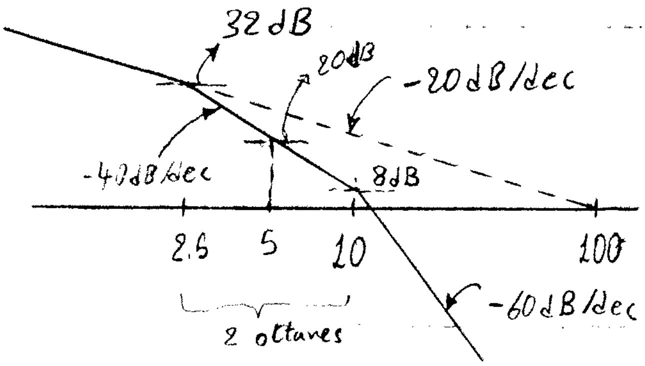
☆ to have $K_v = 100\ sec^{-1}$ $\therefore \lim_{S'\to 0} S' G(S') = 100$.

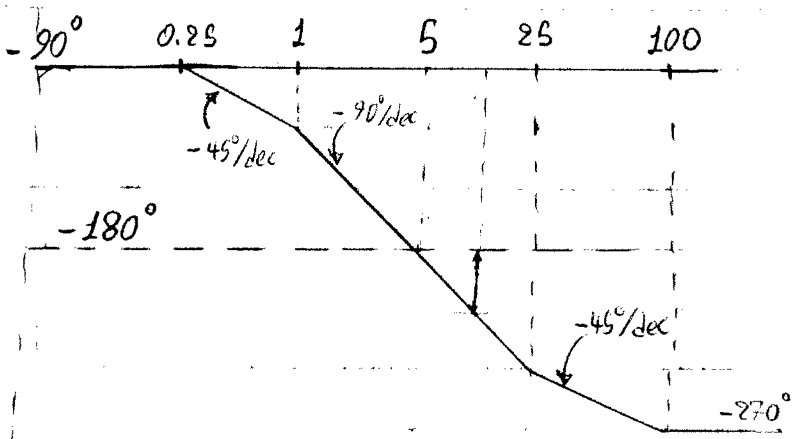
$\therefore K = 100$

☆ Drawing the bode plot of $G(S') = \dfrac{100}{S'(1+S'/10)(1+S'/2.5)}$

Corner frequencies @ $\omega = 10$ rad/sec

& $\omega = 2.5$ rad/sec





$\omega_{gc} = 13.6$ rad/sec

$GM = -20$ dB

$\omega_{pc} = 5$ rad/sec

$PM = -39°$

System is Unstable

☆ We will use a lead-lag compensator to achieve the desired 45° phase margine.

☆ Two Solutions are shown and both will be considered correct.

Solution #1

Assumptions:

* $\omega_{gc_{new}} = \omega_{pc_{old}} = 5$ rad/sec
* $B = \frac{1}{\alpha} = 10$

* We want to calculate τ_{lead}, τ_{lag}

$\frac{1}{\tau_{lag}} = 0.1\,\omega_{gc_{new}} \rightarrow \therefore$ $\tau_{lag} = 2$ sec

$\rightarrow$ gain @ $\omega_{gc_{new}} = a = 20$ dB

$\therefore -a = -20$ dB

$\therefore \frac{1}{\tau_{lead}} = \omega_{gc_{new}}$

$\therefore$ $\tau_{lead} = 0.2$ sec

$$G_c(S) = \frac{(1+2S)\,(1+0.2S)}{(1+20S)\,(1+0.02S)}$$

Check for PM

$PM = 180 + \angle G(j\omega_{gc_{new}})$

$PM_{new} = 34.15°$ ☹

OK!

Solution #2

Assumptions:

* $\omega_{gc_{new}} = \omega_{pc_{old}} = 5$ rad/sec

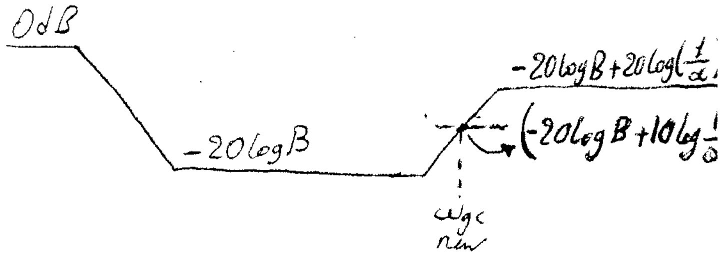
$\alpha \neq 1/B$

Phase @ $\omega_{gc_{new}} = -180°$
to have $PM = 45°$
we need

$\emptyset = 45 + \underset{\text{correction}}{12} = 57° >$

we will take $\alpha = 0.1$
$\hookrightarrow \emptyset \simeq 55°$
still OK

gain @ $\omega_{gc_{new}} = a = 20$ dB



$a - 20\log B + 10\log\left(\frac{1}{\alpha}\right) = 0$ dB

$\therefore$ $B = 31.623$ rad/sec

$\tau_{lag} = \frac{1}{0.1\,\omega_{gc_{new}}} = 2$ sec

$\frac{1}{\tau_{lead}\sqrt{\alpha}} = \omega_{gc_{new}} \longrightarrow$ $\tau_{lead} = 0.6325$ sec

$$G_c(S) = \frac{(1+2S)\quad(1+0.6325S)}{(1+63.246S)\,(1+0.06325S)}$$

$PM_{new} = 49.37°$ ☺

OK!