CAIRO UNIVERSITY **FACULTY OF ENGINEERING**
ELECTRONICS & COMMUNICATIONS DEP. **3rd YEAR, 2010/2011**
CONTROL ENGINEERING

SHEET 6

Root Locus Method

[1] Sketch the root locus for the following transfer functions for the positive values of k:

a) $G(s)H(s) = \dfrac{k}{(s+1)(s+2)}$

b) $G(s)H(s) = \dfrac{ks}{(s+1)^2}$

c) $G(s)H(s) = \dfrac{k(s+2)}{(s+1)(s^2+6s+10)}$

In each of the above cases determine the range of values of k for which the closed loop system is stable.

[2] A unity feedback control system has a forward transfer function:

$$G(s) = \frac{k}{s(1+0.02s)(1+0.01s)}$$

a) Plot the root locus for the positive values of k
b) Determine the value of k that just makes the system unstable.
c) From the root locus plot, determine the value of k for a damping coefficient $\zeta=0.5$.

[3] For a unity feedback system with: $G(s) = \dfrac{k}{s(s+1)(s+2)}$

a) Draw the root locus plot.
b) What value of k should be chosen to have a steady state error of 0.5 to a unit ramp input?
c) What is the minimum value of steady state error to a ramp input which can be achieved with this system?
d) For what values of k will the closed loop system have approximately 15% overshoot for a step input?

[4] Sketch the root locus for a system with an open loop transfer function given by:

$$G(s) = \frac{k}{s^2(s+2)}$$

Show that the system is unstable for all positive values of k. This System can be stabilized by adding a zero on the negative real axis to modify G(s) into G_1 (s) given by:

$$G(s) = \frac{k(s+a)}{s^2(s+2)}$$

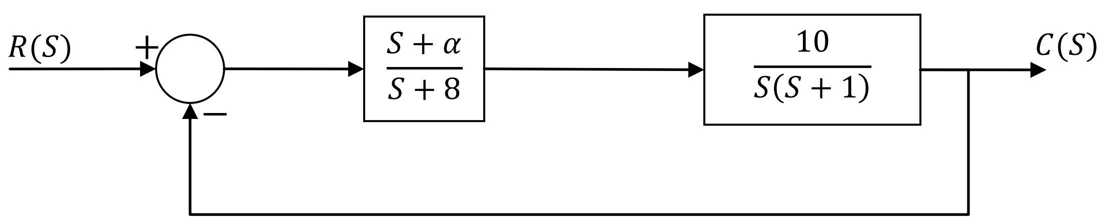
Find the limit on a for stability.

[5] A feedback control system has an open loop transfer function:

$$G(s)H(s) = \frac{k}{s(s+3)(s^2+2s+2)}$$

Plot the root locus as k varies from 0 to ∞.

[6] Consider the system shown in the figure. Sketch the root locus of the system as α varies from zero to infinity. Determine the value of α such that the damping ratio of the dominant closed loop poles is 0.5.

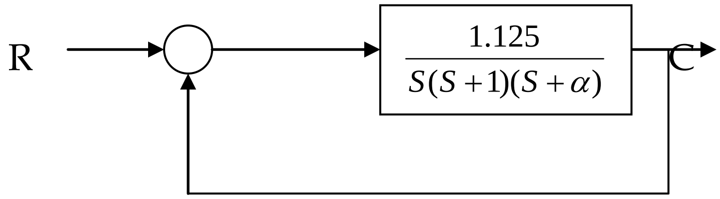


[7] Consider a unity feedback system with a forward path transfer function of:

$$G(s) = \frac{k(s+7)}{(s^2+2s+5)(s+5)}$$

a) Sketch the root locus of the system as k changes from zero to infinity.
b) What is the range of possible values for the damping coefficient ζ of the closed loop?
c) If a series compensator with a transfer function (s+2) is inserted in the above loop, sketch the new root locus (no need to exactly calculate the break in/away points).
d) Repeat part b for the modified system. Comment on your result as related to the properties of PD compensators.

[8] Sketch the root locus of the closed loop system shown in the figure as α varies from 0 to infinity



(Hint: $s^3+s^2+1.125$ has a root at s= -1.5)

[9] (Final2000) For the open loop transfer function $G(S)H(S) = \dfrac{K(S^2+4)}{(S+2)^2(S+5)(S+6)}$

(a) Plot the root locus of the system as K varies from 0 to ∞.

(b) Using this plot, determine the allowable variations in K that constrain the damping ratio of the dominant closed loop poles to the range 0.5 to 0.7.

SUMMARY OF GENERAL RULES FOR CONSTRUCTING ROOT LOCI

1. **Locate the poles (n) and zeros (m) of G(s)H(s) on the s plane.**

   The root-locus branches equal n, start from open-loop poles and terminate at zeros (m branches to m finite zeros, and the remaining n-m branches terminate at infinity).

2. **Determine the root loci on the real axis.**

   complex-conjugate poles and zeros of the open-loop transfer function have no effect on the root-loci location on the real axis

   choose any test point on the real axis, if the total number of real poles and real zeros to the right of the test point is odd ,then this point lies on the root locus.

3. **Determine the asymptotes of the root loci on the real axis.**

   The equation of the asymptotes is given by the real-axis intercept and asymptote angle as follows:

$$\sigma_a = -\frac{(p_1 + p_2 + \ldots + p_n) - (z_1 + z_2 + \ldots + z_m)}{n-m}$$

$$\theta_a = -\frac{(2k+1)\pi}{n-m} \qquad k = 0, \pm 1, \pm 2, \ldots$$

4. **Find the breakaway and break-in points.**

   If there is two adjacent open loop poles or zeros on the real axis, then there **<u>always</u>** exists a breakaway or a break-in point between them respectively.

   If there are two adjacent open loop pole and zero on the real axis, then there **<u>may</u>** exist both breakaway and break-in points between them

   Suppose that the characteristic equation is given by

$$B(s) + KA(s) = 0$$

The breakaway and break-in points can be determined from the roots of

$$\frac{dK}{ds} = \frac{B'(s)A(s) + B(s)A'(s)}{A^2(s)} = 0$$

But not all roots of this equation are breakaway/break-in points:

- If the point lies on the root locus portion of the real axis, then it is an actual point.
- If the point does not lie on the root locus portion of the real axis, then it is not an actual point.
- If two roots are a complex conjugate pair (S=S1 and S=-S1), then the two points are an actual points iff the value of K corresponding to a root S = S1 of $\frac{dK}{ds} = 0$ is positive.

5. **Determine the angle of departure (angle of arrival) of the root locus from a complex pole (at complex zero).**

   Angle of departure from a complex pole = 180
   – (sum of the angles of vectors to a complex pole from other poles)
   + (sum of the angles of vectors to a complex pole from zeros)

   Angle of arrival at a complex zero = 180
   – (sum of the angles of vectors to a complex zero from other zeros)
   + (sum of the angles of vectors to a complex pole from poles)

6. **Find the points where the root loci may cross the imaginary axis.**

a) Use Routh's stability criterion

b) Let s=jw in characteristic equation, equate both the real part and imaginary part to zero, and then solve for w_{osc} and K.

7. **The value of K corresponding to any point S on a root locus can be obtained from** $K = \frac{\text{product of lengths between point s to poles}}{\text{product of lengths between point s to zeros}}$, and the lengths can be evaluated graphically from the root locus sketch.