[6](Final 2004)A unity feedback system has $G(S)=\frac{10}{S(1+10S)}$:

(a) Design a series compensator that will eliminate the steady state error for a unit ramp input signal. The resulting closed loop system should have PM=30.

Solution:

In order to eliminate the steady state error for a unit ramp, we need to increase the system type. So we need a PI controller:

$$G_c(s)=\frac{K(1+TS)}{S}$$

So the complete open-loop system will be:

$$G_{ol}=\frac{10K(1+TS)}{S^2(1+10S)}$$

We need to analyze the system to get the values of K and T that will make the system have a PM=30°.

As this system is type 2, we need the pair of pole and zero to give a positive phase, to ensure the stability of the system, or in other words we need the pole and zero to act as a **lead network** (T>10).

The phase of this open-loop T.F. will have an asymptotic start and end values equal to -180°. We will design the system to have its gain cross over frequency at the maximum positive phase of the lead network $\omega_{gc}=\frac{1}{T_{lead}\sqrt{\alpha}}$.

In order to have a PM=30°, we need to have $\varphi=30°$.

$$\alpha=\frac{1-\sin\varphi}{1+\sin\varphi}=\frac{1}{3}$$

But we have $\alpha T_{lead}=10$

$$\therefore T_{lead}=T=30\ sec.$$

$$\therefore \omega_{gc}=\frac{1}{10\sqrt{3}}rad/sec.$$

$$\because \left|G_{ol}\left(j\omega_{gc}\right)\right| = 1$$

$$\left|\frac{10K\left(1 + j\frac{30}{10\sqrt{3}}\right)}{\left(\frac{j}{10\sqrt{3}}\right)^2\left(1 + j\frac{10}{10\sqrt{3}}\right)}\right| = 1$$

$$\frac{10K \times 2}{\frac{1}{300} \times \frac{2}{\sqrt{3}}} = 1$$

$$K = \frac{1}{3000\sqrt{3}}$$

$$\therefore G_c(S) = \frac{(1 + 30S)}{3000\sqrt{3}S}$$