# Quiz 1 Solution

a)

$$G(S) = \frac{2 * 25}{S(S^2 + 8S + 25)}$$

Corner frequencies

Magnitude

$\omega_n = 5$ rad/sec

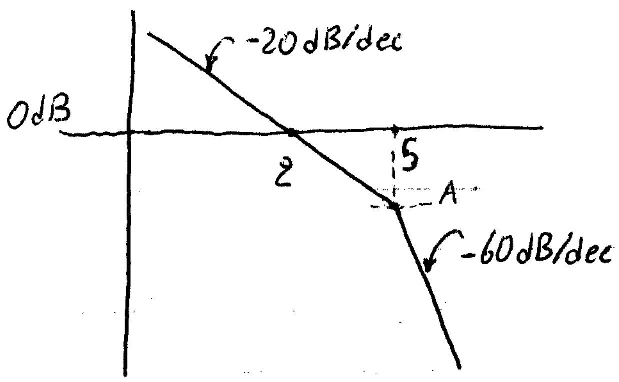
having $\zeta = 0.8$

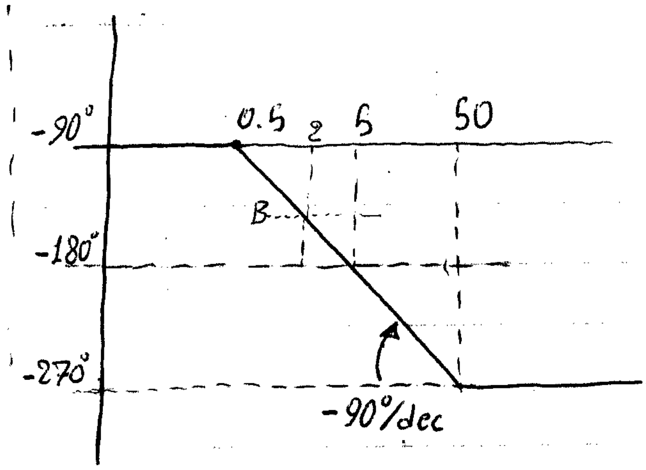
$\because \zeta > \frac{1}{\sqrt{2}}$

$\therefore$ No peaking

Phase

$\omega$ : 0.5 → 50 rad/sec




b) $\omega_{gc} = 2$ rad/sec

$\omega_{pc} = 5$ rad/sec

GM = -A where A is the magnitude @ $\omega_{pc}$

$$A - 0 = -20 \log\left(\frac{5}{2}\right) = -7.96 \text{ dB}$$

$\therefore$ **GM = 7.96 dB**

PM = 180 + B where B is the phase @ $\omega_{gc}$

$$B + 90 = -90 \log\left(\frac{2}{0.5}\right) \rightarrow B = -144.2°$$

**PM = 35.8°**

c) $GM_{new} = GM_{old} - 20 \log k$

$20 = 7.96 - 20 \log k$ $\therefore$ $k = 0.25$

d) to have a $PM = 45^\circ$ we need $\omega_{gc}$ to ~~satisfy~~ have a phase $45^\circ - 180^\circ = 135^\circ$

$\therefore$ calculating the new $\omega_{gc}$

$-135 - (-90) = -90 \log\left(\frac{\omega_{gc_{new}}}{0.5}\right)$ $\therefore$ $\omega_{gc_{new}} = 1.58$ rad/sec

then we need $k$ that makes → Magnitude @ $\omega_{gc_{new}} = C$ where

$C - 0 = -20 \log\left(\frac{\omega_{gc_{new}}}{2}\right)$ $\therefore$ $C = 2.04$ dB

note that increasing $k$ will increase $\omega_{gc}$ & decrease the PM & GM

$C + 20 \log k = 0$ dB → $\therefore$ $k = 0.79$ $\therefore$ $k \leq 0.79$

and also $k > 0$

e) to have $t_s = 4$ sec $= \frac{4}{\zeta\omega_n}$ → of the closed loop T.F.

$\therefore \zeta\omega_n = 1$, then we get the characteristic equation

$$S^3 + 8S^2 + 25S + 50K = 0$$

and compare it with $(S+\alpha)(S^2 + 2\zeta\omega_n S + \omega_n^2)$

$$S^3 + (\alpha + 2\zeta\omega_n)S^2 + (\omega_n^2 + 2\zeta\omega_n\alpha)S + \alpha\omega_n^2 = 0$$

* $\alpha + 2\zeta\omega_n = 8$ → $\alpha = 6 > 5\zeta\omega_n$ ✓
* $\omega_n^2 + 2\zeta\omega_n\alpha = 25$ → $\omega_n = \sqrt{13}$ → $\zeta = \frac{1}{\sqrt{13}}$
* $\alpha\omega_n^2 = 50K$ → ~~$K = 1.74$~~ $K = 1.56$

f) $e_{ss} = \frac{1}{K_v}$, $K_v = 2K$ $\therefore$ ~~$e_{ss} = 0.287$~~ $e_{ss} = 0.3205$

$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 40.4\%$