Antenna Engineering
Lecture 8: Antenna Arrays

ELCN405 – Spring 2011

Communications and Computer Engineering Program

Faculty of Engineering – Cairo University

Outline

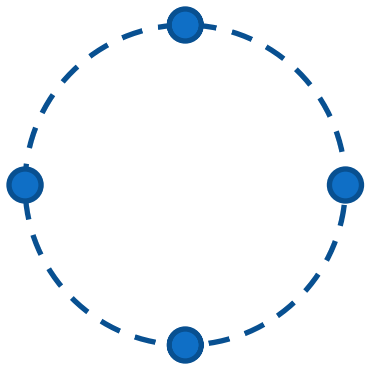
1. Array of Isotropic Radiators
   - Array Configurations
   - The Space Factor
2. The Pattern Multiplication Principle
3. Beam Steering
   - Beam Steering (Scanning) Techniques
   - Electronic Scanning
   - Frequency Scanning
4. Array Feeding Networks

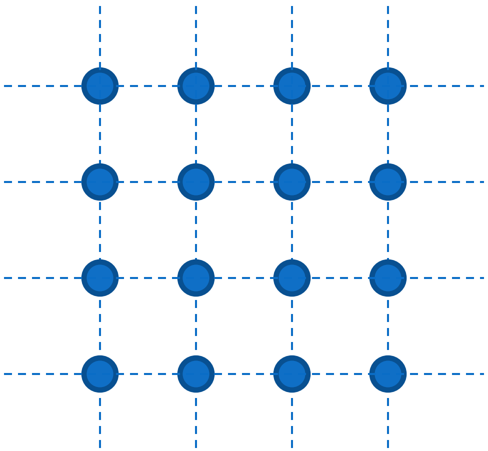
Outline

1. Array of Isotropic Radiators
   - Array Configurations
   - The Space Factor
2. The Pattern Multiplication Principle
3. Beam Steering
   - Beam Steering (Scanning) Techniques
   - Electronic Scanning
   - Frequency Scanning
4. Array Feeding Networks

Array Configurations

Some Array Configurations





Antenna arrays are arrangements of **identical** antennas in arbitrary configurations. What is the motivation of using antenna arrays?

Space Factor

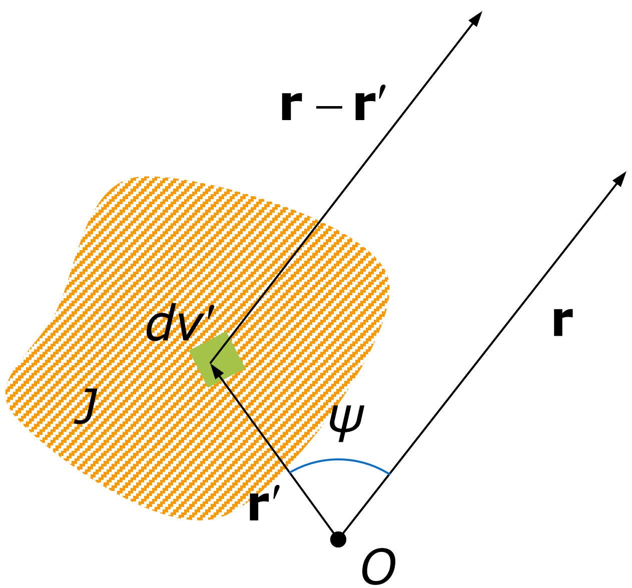
Space Factor for Continuous and Discrete Source Distributions

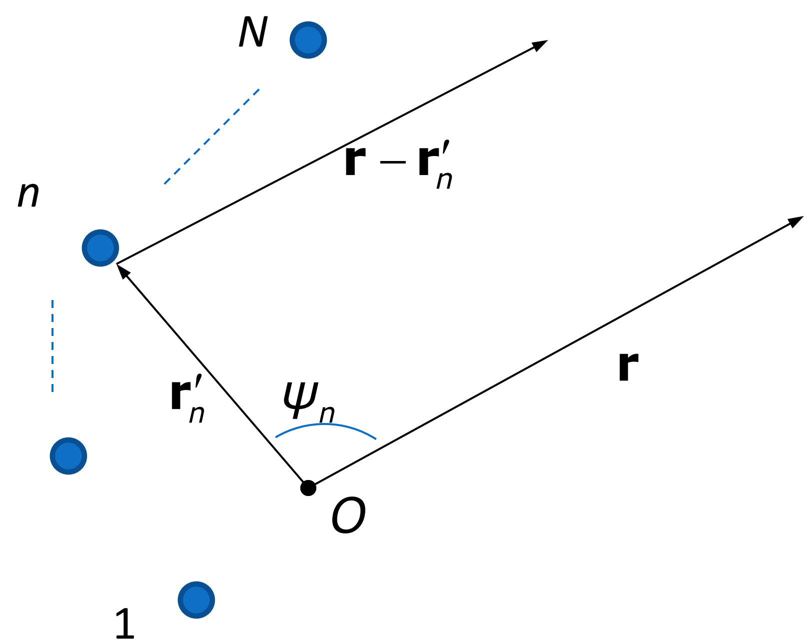
Space Factor (Continuous Distribution)

$$E_{rad} \propto \int_V J(\mathbf{r}') e^{+jkr'\cos\psi} dv'$$

Space Factor (Discrete Distribution)

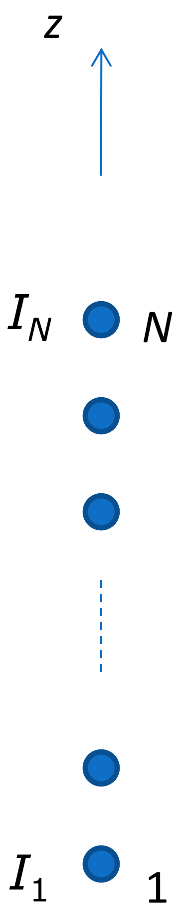
$$E_{rad} \propto \sum_{n=1}^{N} I_n e^{jkr'_n \cos\psi_n}$$





Space Factor

Space Factor for Linear Arrays



Array on *Z*-Axis

$$SF=\sum_{n=1}^{N}\left|I_n\right|e^{j\varphi_n}e^{jkr_n'\cos\psi_n}=\sum_{n=1}^{N}\left|I_n\right|e^{j\varphi_n}e^{jkz_n\cos\theta}$$

$$=\sum_{n=1}^{N}\left|I_n\right|e^{-j(n-1)\beta}e^{jk(n-1)d\cos\theta}$$

Progressive Phase Shift

$$\varphi_n=-(n-1)\beta$$

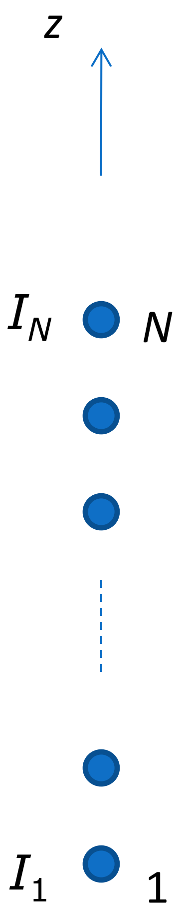
Equidistant Elements

$$z_n=(n-1)d$$

$$SF=\sum_{n=1}^{N}e^{j(n-1)(kd\cos\theta-\beta)}=\frac{1-e^{jN(kd\cos\theta-\beta)}}{1-e^{j(kd\cos\theta-\beta)}}$$

Space Factor

Space Factor for Uniform Linear Array



$$\left|SF(\theta)\right| = \left|\frac{\sin\left(N\frac{kd}{2}(\cos\theta - \cos\theta_0)\right)}{\sin\left(\frac{kd}{2}(\cos\theta - \cos\theta_0)\right)}\right|$$

$$\beta = kd\cos\theta_0$$

$$\left|SF(\psi)\right| = \left|\frac{\sin\left(N\frac{kd}{2}(\cos\psi - \cos\psi_0)\right)}{\sin\left(\frac{kd}{2}(\cos\psi - \cos\psi_0)\right)}\right|$$

$$\beta = kd\cos\psi_0$$

The cosine can be replaced by sine in the space factor expression if the angle is measured from the normal to the array axis.

Space Factor

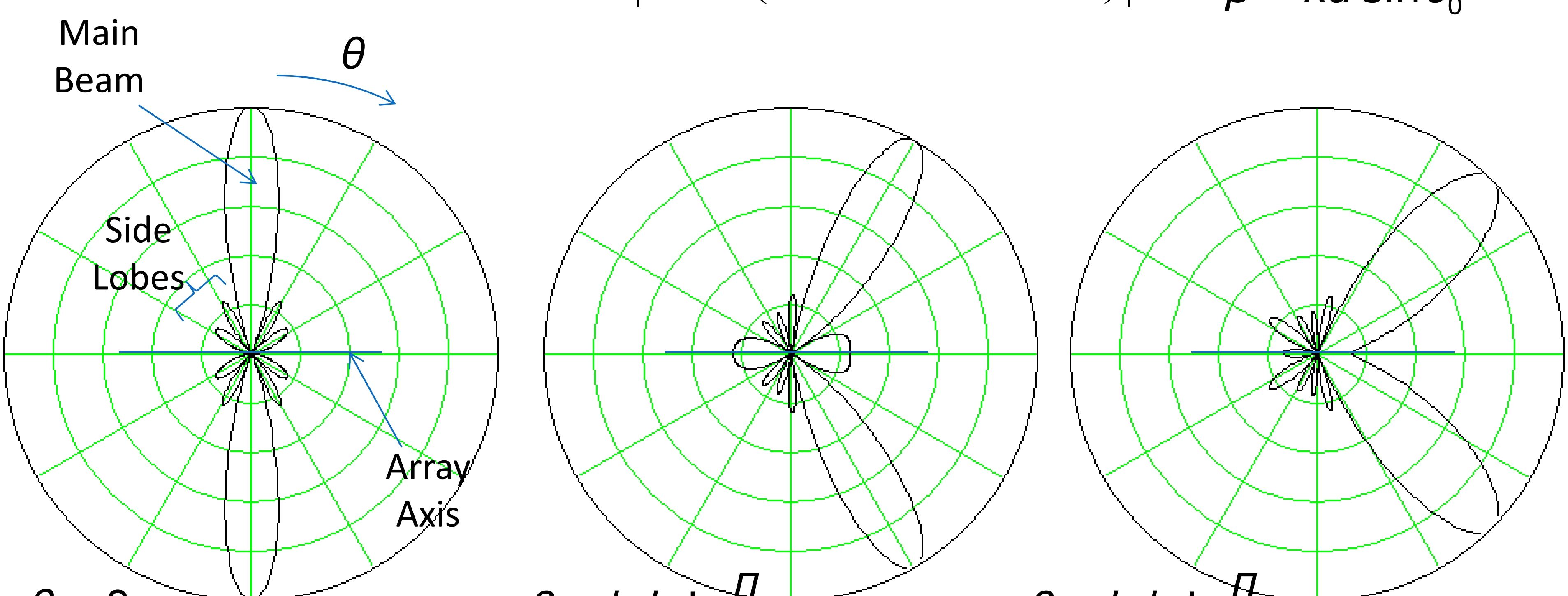
Example

Measured from the normal

Normalized

$$\left|SF_n(\theta)\right| = \left|\frac{\sin\left(N\frac{kd}{2}(\sin\theta - \sin\theta_0)\right)}{N\sin\left(\frac{kd}{2}(\sin\theta - \sin\theta_0)\right)}\right|$$

$$\beta = kd\sin\theta_0$$



$\beta = 0$ Broadside Radiation

$\beta = kd\sin\frac{\pi}{6}$

$\beta = kd\sin\frac{\pi}{4}$

$kd = \pi$

$N = 6$

Space Factor

Main Parameters

$$\left|SF_n(\theta)\right| = \left|\frac{\sin\left(N\frac{kd}{2}(\sin\theta - \sin\theta_0)\right)}{N\sin\left(\frac{kd}{2}(\sin\theta - \sin\theta_0)\right)}\right|$$

Main beam: at the direction determined by the progressive phase shift.

Special cases: Broad-side and end-fire.

Nulls: when the numerator goes to zero, but the denominator is not zero.

Side lobes: Occur half-way between two nulls (excluding the main beam).

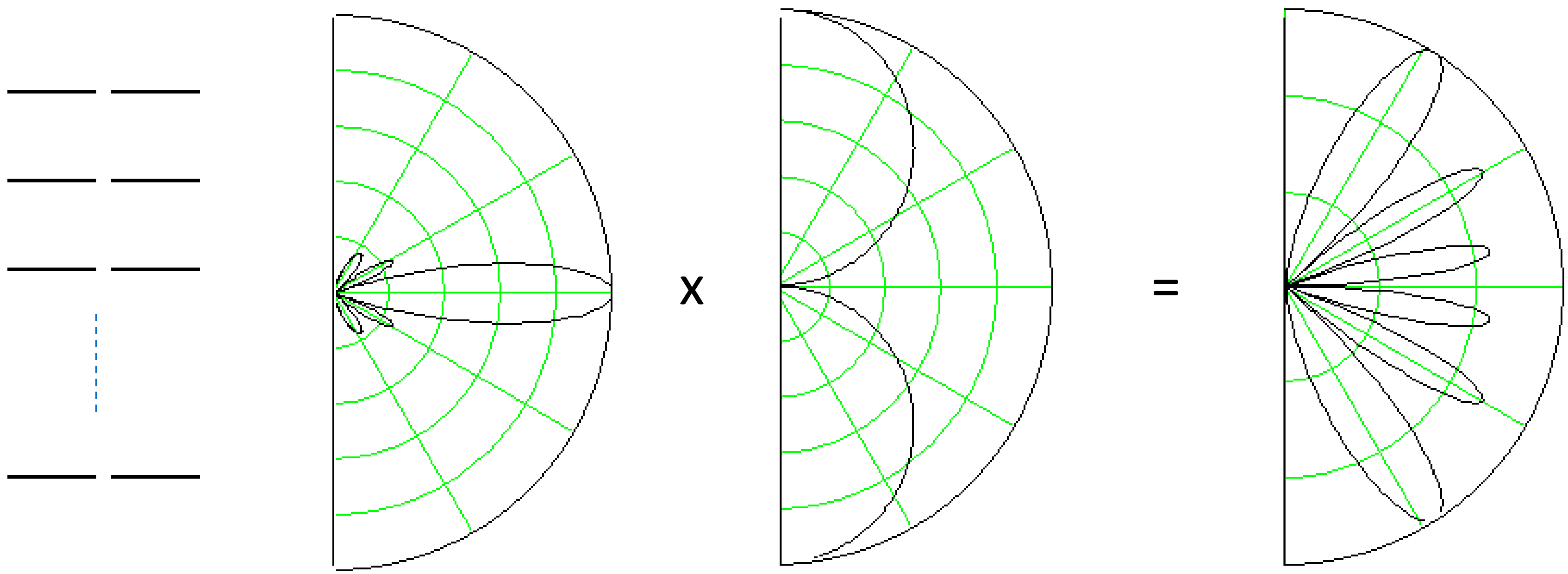
Side lobe level (SLL): Prove that the first side-lobe level for a sufficiently large array is approximately 13 dB (below the main beam level).

Outline

Pattern Multiplication

Total Radiation Pattern of Array of Known Antenna Element

Radiation Pattern = Element Pattern x Array Factor



The pattern multiplication principle neglects the effect of mutual coupling (mutual impedance) between elements, i.e., it assumes that the elements are isolated. Discuss these assumptions.

Outline

1. Array of Isotropic Radiators
   - Array Configurations
   - The Space Factor
2. The Pattern Multiplication Principle
3. Beam Steering
   - Beam Steering (Scanning) Techniques
   - Electronic Scanning
   - Frequency Scanning
4. Array Feeding Networks

Beam Steering Methods

Mechanical, Electronic and Frequency Scanning

1. **Mechanical Scanning**: The beam is steered by physically rotating the whole antenna/antenna array.

2. **Electronic Scanning**: The beam is steered by electronically changing the relative phases of the feeding currents of the array elements.

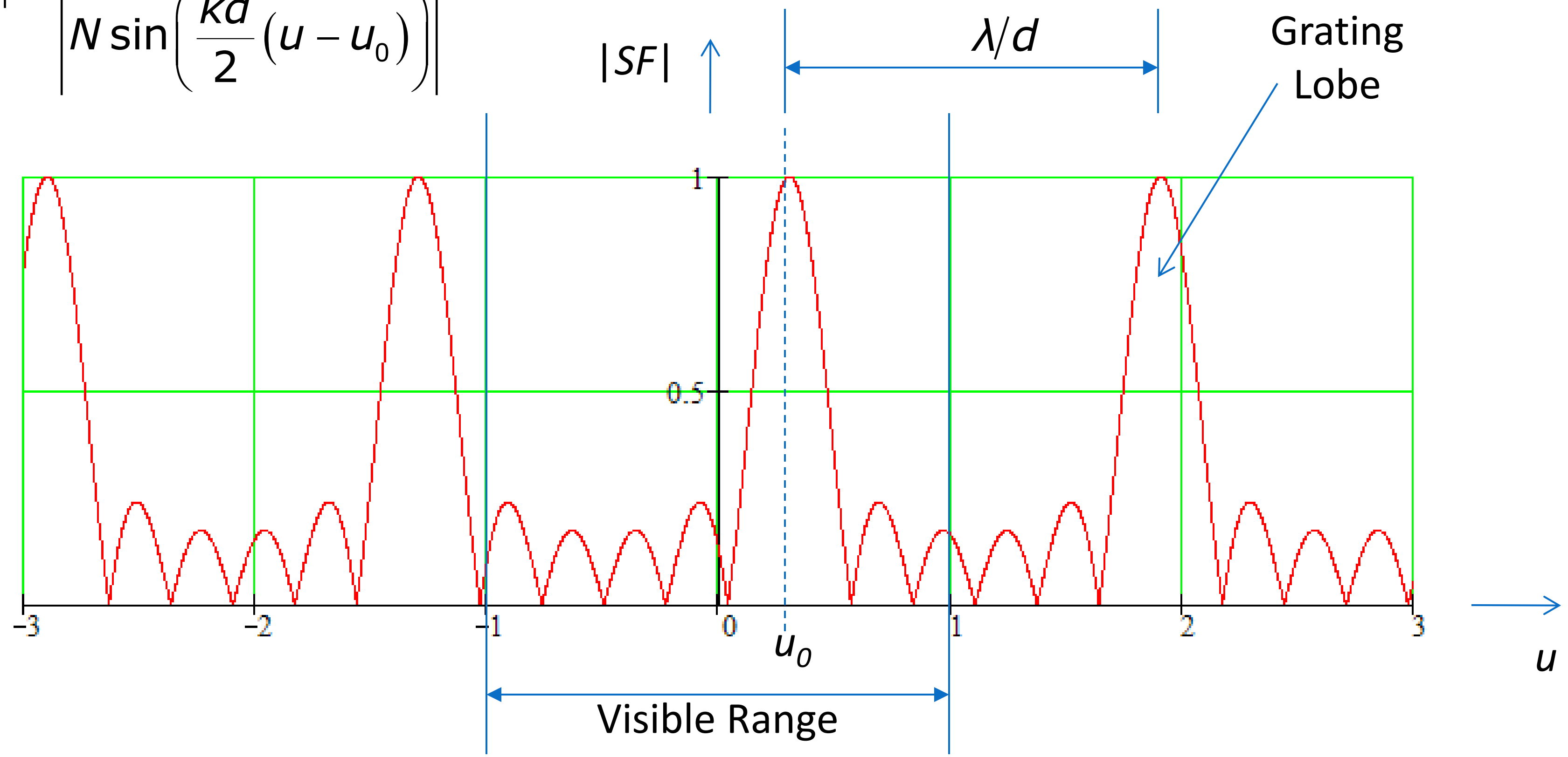
3. **Frequency Scanning**: The beam is steered by changing the operating frequency of the source, which in turn results in changing the relative phases of the element feeding current.

What are the advantages and disadvantages of each?

Electronic Scanning

Visible Range and Maximum Scan Angle

$$|SF_n| = \left| \frac{\sin\left(N \frac{kd}{2}(u-u_0)\right)}{N \sin\left(\frac{kd}{2}(u-u_0)\right)} \right|$$

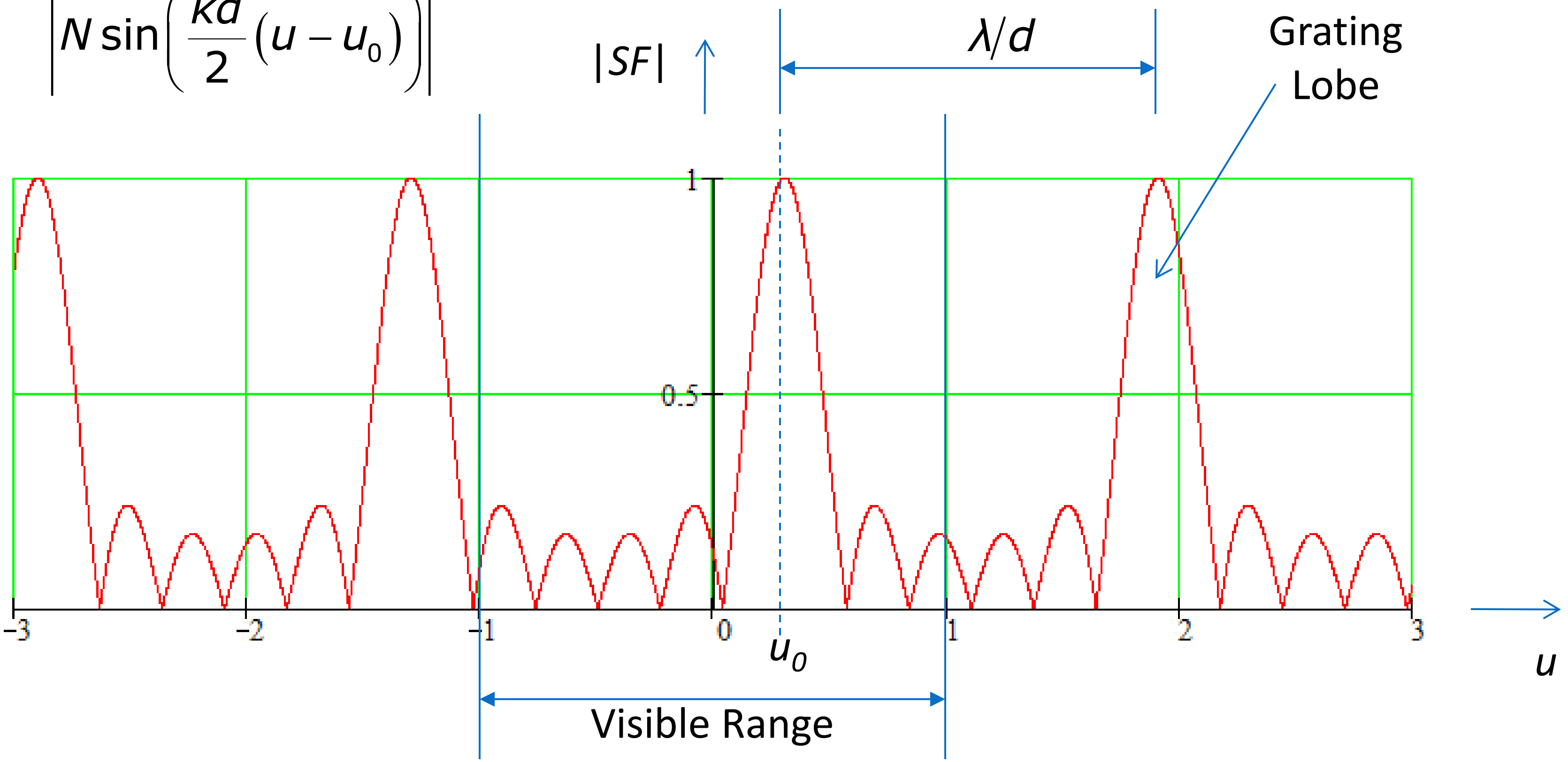


$$u_{0max} + 1 = \frac{\lambda}{d} \Rightarrow \boxed{\sin\theta_{0max} = \frac{\lambda}{d} - 1}$$

Electronic Scanning

Visible Range and Inter-Element Spacing

$$|SF_n| = \left| \frac{\sin\left(N \frac{kd}{2}(u-u_0)\right)}{N \sin\left(\frac{kd}{2}(u-u_0)\right)} \right|$$



$$u_{0max} + 1 < \frac{\lambda}{d} \Rightarrow \boxed{d < \frac{\lambda}{u_{0max}+1}}$$

Electronic Scanning

Example

Example

1. Determine the inter-element spacing for a linear array that will allow steering the beam arbitrarily without the appearance of grating lobes.
2. Determine the inter-element spacing for broadside radiation of a linear array to have only one main beam (no grating lobes).

Solution:

Electronic Scanning

Example

Example

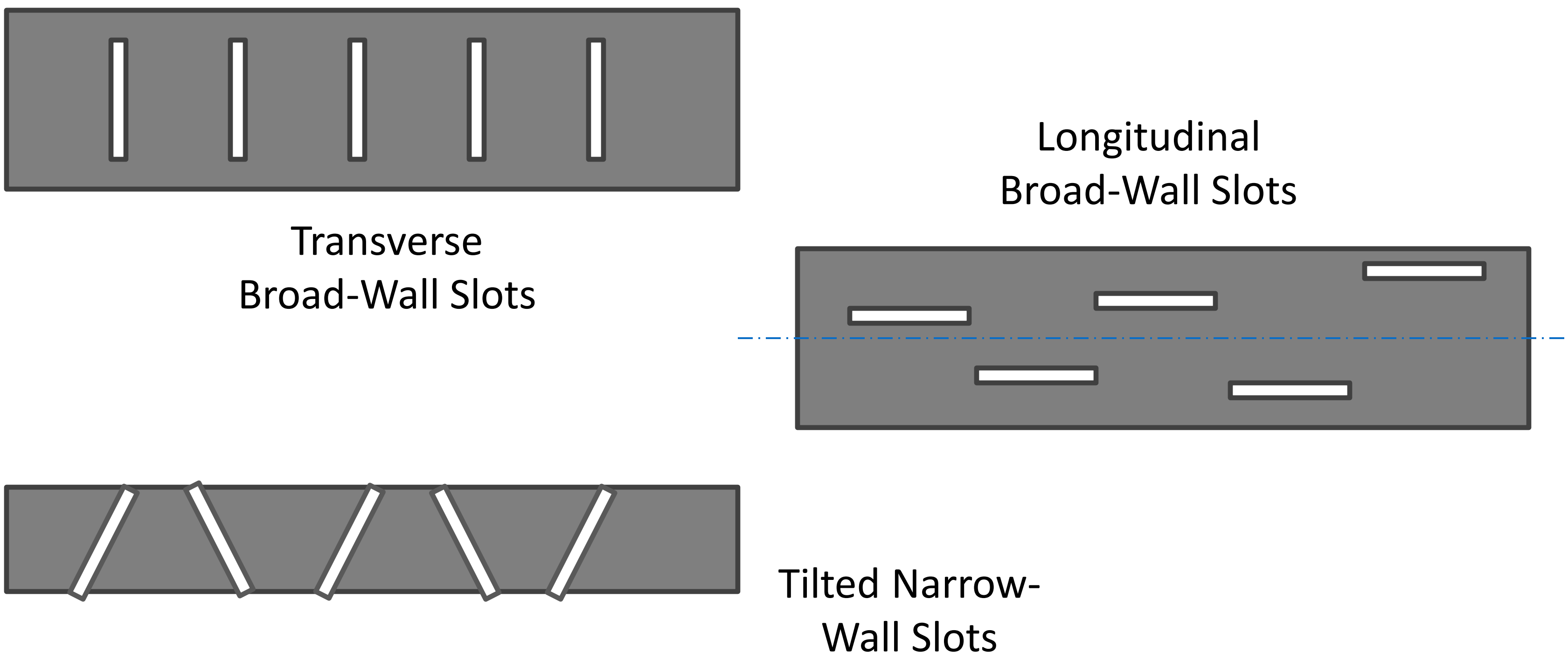
Find the space factor in the following cases (use conventional coordinate system):

- Two equal in-phase sources at (0, 0, 0) and (*d*, 0, 0).
- Two equal in-phase sources at (0, 0, 0) and (*d*, *d*, 0).
- Four equal in-phase sources at (0, 0, 0), (*d*, 0, 0), (0, *d*, 0) and (*d*, *d*, 0).
- Three in-phase sources at (0, 0, 0), (0, 0, *d*) and (0, 0, 2*d*) with ratio of magnitudes 1:2:1.

Solution:

Frequency Scanning

Slotted Waveguide Antenna Arrays



Waveguide slots are excited if they intercept surface current (tangential to the magnetic field).

Explain how frequency scanning works in slotted waveguide arrays?

Frequency Scanning

Example

Example

Design a slotted X-band waveguide antenna array at 9 GHz using:

a. Transverse slots for end-fire radiation.

b. Longitudinal slots for broad-side radiation.

Neglect the coupling between elements and assume the power reaching all elements is unchanged. Discuss the previous assumptions.

The design should be done under the following conditions:

a. The waveguide is terminated in matched load.

b. The waveguide is terminated in short-circuit termination.

Solution:

Outline

1. Array of Isotropic Radiators
   - Array Configurations
   - The Space Factor
2. The Pattern Multiplication Principle
3. Beam Steering
   - Beam Steering (Scanning) Techniques
   - Electronic Scanning
   - Frequency Scanning
4. Array Feeding Networks

Conclusion

- Antenna arrays may be used to boost the gain of the radiating system and provide control on the beam direction.
- Antenna arrays play an important role in radar systems (electronic scanning) and many communication systems.
- A compact and low-loss feeding network is one of the design challenges in array design. The feeding network may limit the system bandwidth (how?)
- In some arrays, not all the elements are driven; some elements are excited parasitically (e.g. Yagi-Uda array).