Topic 6

Receiving Antennas

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Outline

Lorentz Reciprocity Theorem

Let $\mathbf{E}_1$, $\mathbf{H}_1$, $\mathbf{J}_1$ and $\mathbf{E}_2$, $\mathbf{H}_2$, $\mathbf{J}_2$ be two sets of solutions to Maxwell's equations.

$$\nabla \times \mathbf{E}_1 = -j\omega\mu\mathbf{H}_1 \qquad \nabla \times \mathbf{E}_2 = -j\omega\mu\mathbf{H}_2$$

$$\nabla \times \mathbf{H}_1 = j\omega\varepsilon\mathbf{E}_1 + \mathbf{J}_1 \qquad \nabla \times \mathbf{H}_2 = j\omega\varepsilon\mathbf{E}_2 + \mathbf{J}_2$$

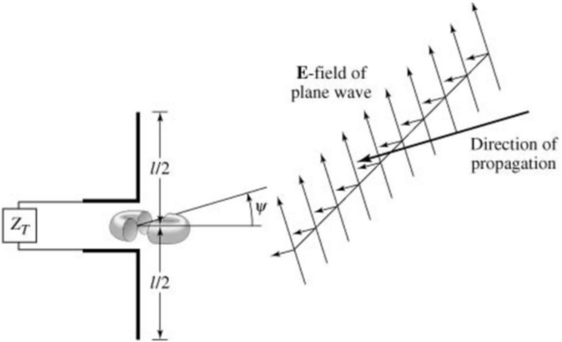
$$\begin{aligned} \nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) &= \nabla \times \mathbf{E}_1 \cdot \mathbf{H}_2 - \mathbf{E}_1 \cdot \nabla \times \mathbf{H}_2 \\ &\quad - \nabla \times \mathbf{E}_2 \cdot \mathbf{H}_1 + \mathbf{E}_2 \cdot \nabla \times \mathbf{H}_1 \\ &= \mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1 \end{aligned}$$

$$\boxed{\oint_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{s} = \int_V (\mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1)\, dv}$$

The reciprocity theorem can be used to provide important concepts such as the antenna *vector effective length* and *antenna effective area.*

Outline

Vector Effective Length



The radiation electric field **E** can be written in terms of the input current as,

$$\mathbf{E}_a = j\eta \frac{kI_{in}e^{-jkr}}{4\pi r}\boldsymbol{\ell}_e$$

where $\boldsymbol{\ell}_e$ is the vector effective length, $\boldsymbol{\ell}_e = \ell_\theta \hat{\mathbf{a}}_\theta + \ell_\phi \hat{\mathbf{a}}_\phi$.
The open-circuit voltage V_{oc} of the receiving antenna,

$$\boxed{V_{oc} = \mathbf{E}_i \cdot \boldsymbol{\ell}_e}$$

Polarization Mismatch

$$V_{oc} = \mathbf{E}_i \cdot \boldsymbol{\ell}_e$$

The open circuit voltage V_{oc} is maximized when the dot product is maximized.

Definition

Polarization efficiency (Polarization mismatch factor): the ratio of the power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density and direction of propagation, whose state of polarization has been adjusted for a maximum received power.

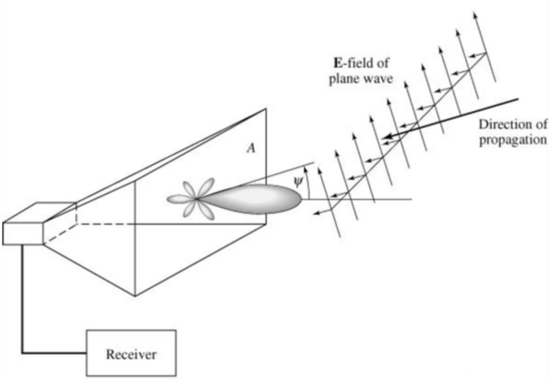
Polarization Mismatch

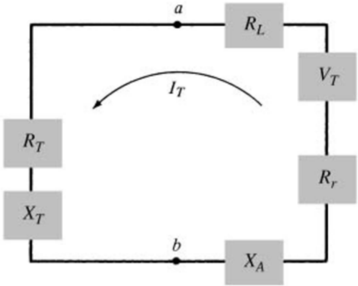
Polarization efficiency (Polarization Loss Factor PLF) $$p = \frac{\left|\mathbf{E}_i \cdot \boldsymbol{\ell}_e\right|^2}{\left|\mathbf{E}_i\right|^2 \left|\boldsymbol{\ell}_e\right|^2}$$

Assuming the electric field of the incoming wave is $\mathbf{E}_i = \hat{\boldsymbol{\rho}}_w E_i$, and the polarization of the electric field of the receiving antenna $\mathbf{E}_a = \hat{\boldsymbol{\rho}}_a E_a$, the Polarization efficiency (Polarization Loss Factor LPF) p,

Polarization efficiency (Polarization Loss Factor PLF) $$p = \left|\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a\right|^2$$

Outline

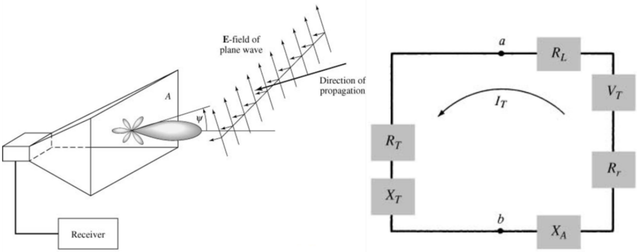




Definition

Effective area (aperture) is the ratio of the *available power* at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being *polarization matched* to the antenna.

Antenna Equivalent Areas



$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i} = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r+R_L+R_T)^2+(X_A+X_T)^2}\right]$$

P_T is maximized to the available power P_A under conjugate matching:
$R_T = R_r + R_L, \qquad X_T = -X_A.$

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r+R_L}\right]$$

Antenna Equivalent Areas

- The *scattering area*: is the equivalent area when multiplied by the incident power density is equal to the scattered or reradiated power, $A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_r + R_L)^2}\right].$
- The *loss area*: is the equivalent area when multiplied by the incident power density is equal to the power dissipated as heat, $A_L = \frac{|V_T|^2}{8W_i}\left[\frac{R_L}{(R_r + R_L)^2}\right].$
- The *capture area*: is the equivalent area when multiplied by the incident power density is equal to the total power captured, $A_c = \frac{|V_T|^2}{4W_i}\left[\frac{1}{R_r + R_L}\right].$

$$A_c = A_{em} + A_s + A_L$$

The received power is half the captured power.

Maximum gain and maximum effective area

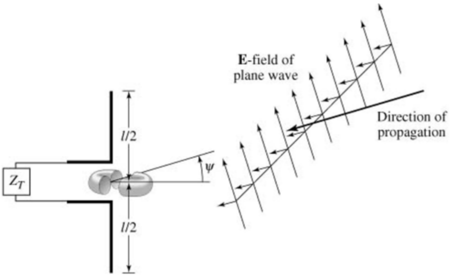
Theorem

The maximum effective area (A_{em}) of any antenna is related to its maximum gain (G_0) by

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} G_0}$$

Received Power from Incident Wave

If the incident power power density is $W_i = \frac{|\mathbf{E}_i|^2}{2\eta}$, and the maximum effective area of the receiving antenna is $A_{em}(\theta_r, \phi_r) = \frac{\lambda^2}{4\pi} G_r(\theta_r, \phi_r)$, then the received power P_r from the receiving antenna,



$$\boxed{P_r = W_i A_{em} p \left(1 - |\Gamma_r|^2\right),}$$

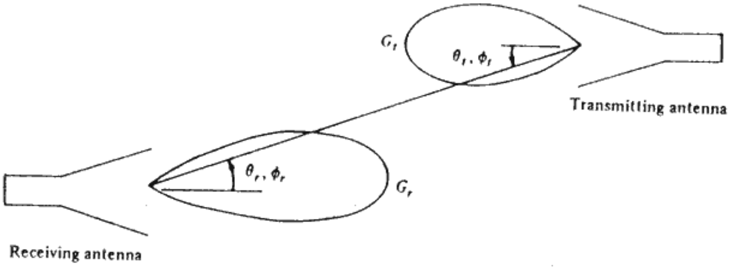
where p is the polarization mismatch factor and the term $\left(1 - |\Gamma_r|^2\right)$ is the impedance mismatch of the receiving antenna, and Γ_r is given by,

$$\Gamma_r = \frac{Z_r - Z_T^*}{Z_r + Z_T}$$

where Z_r is the receiving antenna input impedance.

Outline

Friis Transmission Formula



$$P_{rec} = p\left(1 - |\Gamma_r|^2\right)\left(1 - |\Gamma_t|^2\right)\frac{P_t\lambda_0^2}{16\pi^2 r^2} G_r\left(\theta_r, \phi_r\right) G_t\left(\theta_t, \phi_t\right),$$

where P_t is the *available power* to the transmitting antenna and P_r is the received power in the receiving antenna.

p : polarization mismatch.

$\left(1 - |\Gamma_t|^2\right)$: transmitting antenna impedance mismatch.

$\left(1 - |\Gamma_r|^2\right)$: receiving antenna impedance mismatch.

Examples on Polarization Loss Factor

Sense of rotation

The sense of rotation is always determined by rotating the phase-leading component towards the phase lagging component and observing the field rotation as the wave is viewed as it travels away from the observer.

$$\text{Right-Hand} \equiv \text{Clockwise (CW)}$$

$$\text{Left-Hand} \equiv \text{Counterclockwise (CCW)}$$

A CW circularly polarized uniform plane wave is traveling in the $+z$ direction.
Find the polarization loss factor PLF (dimensionless and in dB) assuming the receiving antenna (in its transmitting mode) is:

1. CW circularly polarized
2. CCW circularly polarized

Examples on Polarization Loss Factor

A linearly polarized uniform plane wave traveling in the $+z$ direction, with a power density of 10 milliwatts per square meter, is incident upon a CW circularly polarized antenna whose gain is 10 dB at I0 GHz. Find the

1. PLF.
2. power (in watts) that will he delivered to a load attached directly to the terminals of the antenna.