Formula Sheet

- Radiation electric field from dipole of length l along z-axis,

$$\mathbf{E} = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right] \hat{\mathbf{a}}_\theta$$

- For infinitesimal dipole of length l ($l \ll \lambda$) along direction $\hat{\mathbf{n}}$,
  - Radiation electric field

$$\mathbf{E} = -j\eta \frac{klI_0 e^{-jkr}}{4\pi r} \left[\hat{\mathbf{n}} - \left(\hat{\mathbf{n}} \cdot \hat{\mathbf{a}}_r\right) \hat{\mathbf{a}}_r \right]$$

  - Radiation Resistance

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2.$$

- Radiation electric field from a small loop of radius a ($2\pi a < \lambda/10$) with its axis oriented along direction $\hat{\mathbf{n}}$,

$$\mathbf{E} = j\eta \frac{k^2 a^2 I_0 e^{-jkr}}{4r} \hat{\mathbf{n}} \times \hat{\mathbf{a}}_r$$

- Directivity of $\lambda/2$ dipole is 1.643 and radiation resistance is 73 Ω.

- Matrix transformation of any vector from Cartesian to spherical coordinates,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

- $$\int e^{\alpha x} \sin(\beta x + \gamma) = \frac{e^{\alpha x}}{\alpha^2 + \beta^2} \left[\alpha \sin(\beta x + \gamma) - \beta \cos(\beta x + \gamma) \right]$$

- Angle ψ between the two directions (θ, ϕ) and (θ', ϕ'),

$$\cos\psi = \sin\theta\sin\theta'\cos(\phi - \phi') + \cos\theta\cos\theta'$$

- Aperture Efficiency ϵ_{ap} is,

$$\epsilon_{ap} = \frac{\left| \iint_{A_p} \mathbf{E}_a ds' \right|^2}{A_p \iint_{A_p} \left|\mathbf{E}_a\right|^2 ds'}$$

- Antenna Vector Effective Length $\boldsymbol{\ell}$,

$$\mathbf{E} = j\eta \frac{kI_{in}}{4\pi r} \boldsymbol{\ell} e^{-jkr}$$