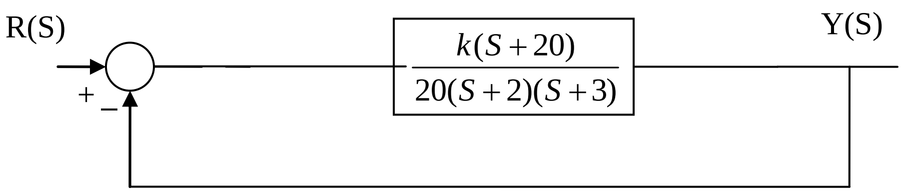
SHEET 5

Lead-Lag Compensation Techniques

[1] For the following system, Design a compensator such that the position error constant of the overall system is 1.667, and the phase margin is greater than or equal 70 degrees and the gain crossover frequency after compensation does not exceed 5 rad\sec.



[2]An uncompensated unity feedback control system whose open loop transfer function is $G(S)=\frac{200}{(1+S)(S+2)}$, is to be designed to meet the following specifications: The gain crossover frequency after compensation does not exceed 4 rad\sec, and Phase Margin PM ≥ 35.

(a) Use a Bode plot to determine the values of ω_{gc}, ω_{pc}, GM and PM for the uncompensated system.

(b) Design the necessary compensator.

(c) What are the resulting values of ω_{gc}, PM and steady-state error e_{ss} for a unit step input.

[3] (Final 1998) An uncompensated unity feedback control system whose open-loop transfer function is $G(S) = \dfrac{24K}{S(S+2)(S+6)}$ is to be designed to meet the following performance specifications:

- When the input is a ramp with slope (velocity) equal 2л rad/sec, the steady state error in position must be ≤ л / 10 rad.
- Phase Margin PM ≥ 45°.
- The new gain crossover frequency $\omega_{gc} \geq 1$ rad/sec.

a) Use the Bode plot to design the necessary compensator.

b) Synthesize your compensator circuit.

c) Determine ω_{gc} and PM for both uncompensated and the resulting compensated systems.

[4] (Final 2001) An uncompensated unity feedback control system whose open loop transfer function is $G(S) = \dfrac{K}{(1+S/3)^3}$ is to be designed to meet the following performance specifications:

$K_p = 4$, GM ≥ 12 db. And PM ≥ 40°.

(a) Use the Bode plot to determine the values of ω_{gc}, ω_{pc}, GM and PM for the uncompensated system.

(b) Design a lead compensator to meet the performance specifications. Avoid any zero-pole cancellation in your design and synthesize your compensator circuit.

(c) What are the resulting values of ω_{gc}, ω_{pc}, GM, PM and steady-state error e_{ss} for a unit step input.

[5] (Final 2000) An uncompensated unity feedback control system whose open loop transfer function is $G(S) = \dfrac{160K}{S(S+8)(S+20)}$, is to be designed to meet the following performance specifications:

(i) Steady state error = 0.01 due to a unit ramp input.

(ii) Gain Margin $\geq$ 10 db.

(iii) Phase Margin PM $\geq$ 40.

(iv) Gain crossover frequency $10 < \omega_{gc} < 20$ rad/sec.

(a) Use a Bode plot to determine the values of ω_{gc}, ω_{pc}, GM and PM for the uncompensated system.

(b) Design the necessary compensator. Avoid any zero-pole cancellation in your design and synthesize your compensator circuit.

(c) What are the resulting values of ω_{gc}, ω_{pc}, GM, PM and steady-state error e_{ss} for a unit step input

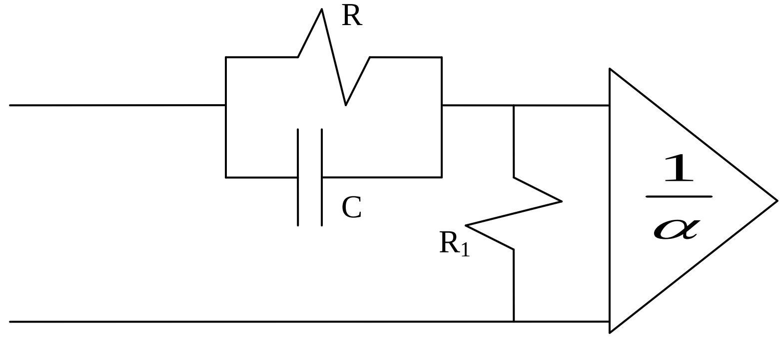
[6](Final 2004)A unity feedback system has $G(S) = \dfrac{10}{S(1+10S)}$:

(a) Design a series compensator that will eliminate the steady state error for a unit ramp input signal. The resulting closed loop system should have PM=30.

Summary

- ***LEAD COMPENSATOR:***

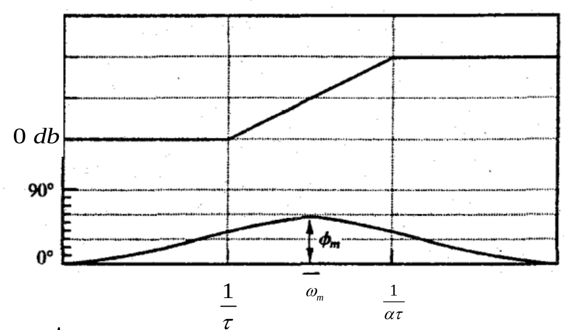
I. Circuit synthesization:



$$G_c(S) = \frac{(\tau S + 1)}{(\alpha \tau S + 1)}$$

Where $\alpha = \frac{R_1}{R_1 + R} < 1, \tau = CR$

II. Bode diagram of lead compensator:



III. Compensation steps:

1. Assume that the open loop transfer function of the compensated system is:
$$G_c(S)G_1(S) = \left[\frac{(\tau S + 1)}{(\alpha \tau S + 1)}\right]\left[KG(S)\right]$$
Determine gain *K* to satisfy the requirement on the given static error constant.

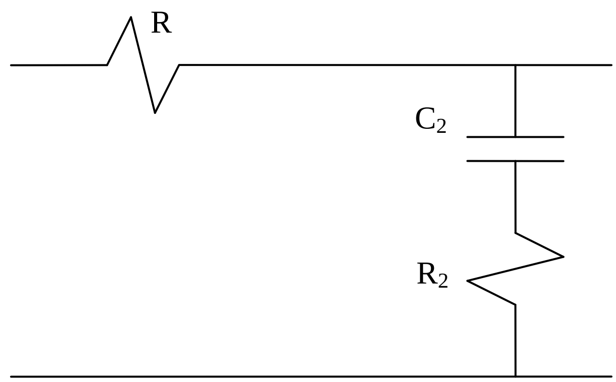
2. Plot $KG(S)$ and calculate from the plot $\omega_{gc\ old}, \omega_{pc\ old}, GM_{old}, PM_{old}$
3. Get $\Phi_m = PM_{req} - PM_{old} + 5^{\circ}$
4. Get α from $\sin \Phi_m = \frac{1 - \alpha}{1 + \alpha}$,then check that $0.1 \le \alpha$<1.
5. Draw the line of magnitude equals $20 \log \sqrt{\alpha}$ on the old plot.
6. The frequency where this line will intersect with the old plot is ω_m
7. Select $\omega_{gc\ new} = \omega_m$.
8. Get τ from $\omega_m = \frac{1}{\tau \sqrt{\alpha}}$.
9. Redraw the magnitude and phase, after compensation, and get $\omega_{gc\ new}, \omega_{pc\ new}, GM_{new}, PM_{new}$
10. If the requirements are not satisfied (specially the PM), return to step 3 and increase the correction term from $5^{\circ} \to 20^{\circ}$.

IV. Comments on lead compensation:

1) Lead compensator essentially yields an improvement in transient response and a small change in the steady state accuracy.
2) It increases the gain crossover frequency [ω_{gc}].
3) It increases the Bandwidth of the system [ω_B], so it may accentuate high frequency noise effects.
4) It increases the resonant frequency [ω_r].
5) It decreases the resonant peak [M_r], which means that it improves the stability.
6) It speeds up the response of the system, so it acts approximately as a PD controller..
7) The maximum phase lead Φ_m shouldn't be larger than 55° (or $\alpha \le 0.1$), because such values will require an additional gain of excessive values, if more than 55 is needed; two lead compensators may be used in series with an isolating amplifier.

- ***LAG COMPENSATOR:***

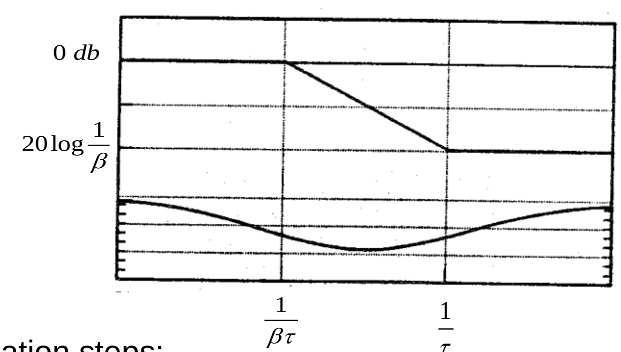
I. Circuit synthesization:



$$G_c(S) = \beta \frac{(\tau S + 1)}{(\beta \tau S + 1)}$$

Where $\beta = \dfrac{R_2 + R}{R_2} > 1, \tau = C_2 R_2$

II. Bode diagram of lag compensator:



III. Compensation steps:

1. Assume that the open loop transfer function of the compensated system is:

$$G_c(S)G_1(S) = \left[\frac{(\tau S + 1)}{(\beta \tau S + 1)}\right]\left[KG(S)\right]$$

Determine gain *K* to satisfy the requirement on the given static error constant.

2. Plot $KG(S)$ and calculate from the plot $\omega_{gc\ old}, \omega_{pc\ old}, GM_{old}, PM_{old}$
3. From $\angle G(S)\big|_{\omega_{gc\ new}} = -180^\circ + PM_{req} + 12^\circ$,you can get $\omega_{gc\ new}$ analytically.

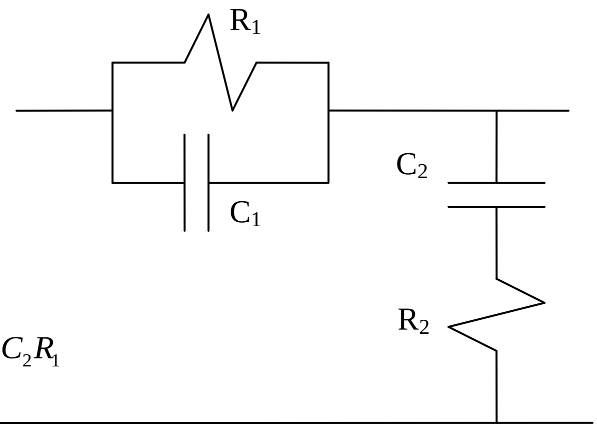
4. Get $\omega = \frac{1}{\tau}$ 1 octave to 1 decade below $\omega_{gc\ new}$ (usually $\omega = \frac{1}{\tau} = 0.1\omega_{gc\ new}$).
5. From plot you can get β ,as $\left|G(S)\right|_{\omega gc\ new}\ in\ db = -20\log\beta$.
6. Redraw the magnitude and phase, after compensation, and get $\omega_{gc\ new}, \omega_{pc\ new}, GM_{new}, PM_{new}$
7. If the requirements are not satisfied (specially the PM), return to step 4 and choose another value for $\omega = \frac{1}{\tau}$.

IV. Comments on lag compensation:

1) Lag compensator essentially yields an improvement in the steady state accuracy at the expense of increasing the transient response time.
2) It decreases the gain crossover frequency [ω_{gc}].
3) It decreases the Bandwidth of the system [ω_B], so it may suppress high frequency noise effects.
4) It decreases the resonant frequency [ω_r].
5) It stabilizes the unstable systems.
6) It speeds down the response of the system, so it acts approximately as a PI controller.

- ***LAG-LEAD COMPENSATOR:***

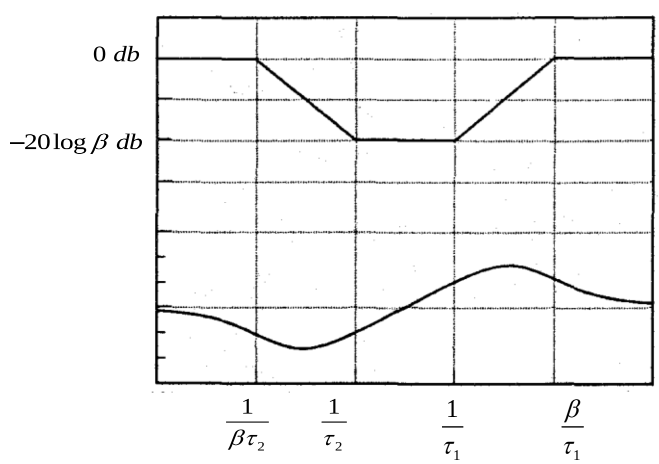
I. Circuit synthesization:



$$G_c(S) = \frac{(\tau_1 S + 1)(\tau_2 S + 1)}{(\frac{\tau_1}{\beta} S + 1)(\beta\tau_2 S + 1)}$$

Where $\tau_1 = C_1 R_1, \tau_2 = C_2 R_2, \frac{\tau_1}{\beta} + \beta\tau_2 = C_1 R_1 + C_2 R_2 + C_2 R_1$

II. Bode diagram of lag compensator:



III. Compensation steps:

1. Assume that the open loop transfer function of the compensated system is:

$$G_c(S)G_1(S) = \left[\frac{(\tau_1 S+1)(\tau_2 S+1)}{(\frac{\tau_1}{\beta} S+1)(\beta\tau_2 S+1)} \right] [KG(S)]$$

Determine gain *K* to satisfy the requirement on the given static error constant.

2. Plot $KG(S)$ and calculate from the plot $\omega_{gc\ old}, \omega_{pc\ old}, GM_{old}, PM_{old}$.
3. Let $\omega_{gc\ new} \approx \omega_{pc\ old}$ and $\beta = 10$ (the distance $a = |G(S)|_{\omega gc\ new}$ *in db must be less than* $20db$).
4. Get $\omega = \frac{1}{\tau_2}$ 1 decade below $\omega_{gc\ new}$ ($\omega = \frac{1}{\tau_2} = 0.1\omega_{gc\ new}$).
5. Draw a straight line of slope 20db/decade, passing through the point ($\omega_{gc\ new}, -a$), the intersection of this line and the -20db line determine the corner frequency $\omega = \frac{1}{\tau_1}$.
6. Redraw the magnitude and phase, after compensation, and get $\omega_{gc\ new}, \omega_{pc\ new}, GM_{new}, PM_{new}$

IV. Comments on lag compensation:

1) Lag-Lead compensator essentially yields an improvement in both the steady state accuracy and the transient response time.