Topic 3

Fundamental Parameters of Antennas

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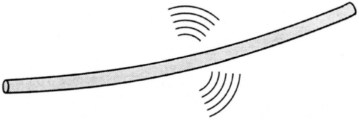
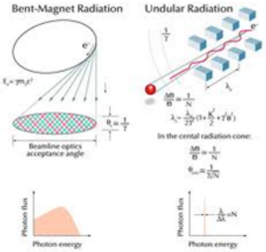
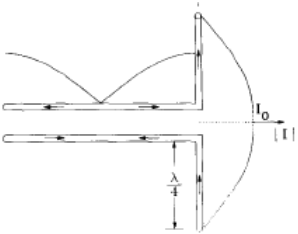
Electromagnetic Radiation

Time-changing current radiates and accelerated charge radiates.







Fundamental Parameters of Antennas

1. Radiation Pattern
   - Radiation Pattern Lobes
   - Isotropic, Directional, and Omnidirectional Pattern
   - Principal Patterns
   - Field Regions
   - Solid Angle
2. Radiation Power Density
3. Radiation Intensity
4. Beamwidth
5. Directivity
6. Beam Solid Angle Ω_A (Beam Area)
7. Antenna Input Impedance and Radiation Efficiency
8. Antenna Gain

Many of the definitions of these terms are taken from the IEEE Std 145-1993.

Outline

Radiation Patterns

Antenna Radiation Pattern or Antenna Pattern

The spatial distribution of a quantity that characterizes the electromagnetic field generated by an antenna.

- Radiation is a spherical TEM fields with propagation in $\hat{\mathbf{a}}_r$ direction and fields in $\hat{\mathbf{a}}_\theta$ and $\hat{\mathbf{a}}_\phi$ directions.
- $|E_\theta|$ and $|E_\phi| \propto 1/r$

- $|E_\theta|$
- $|E_\phi|$
- Phases of these fields, δ_θ and δ_ϕ.

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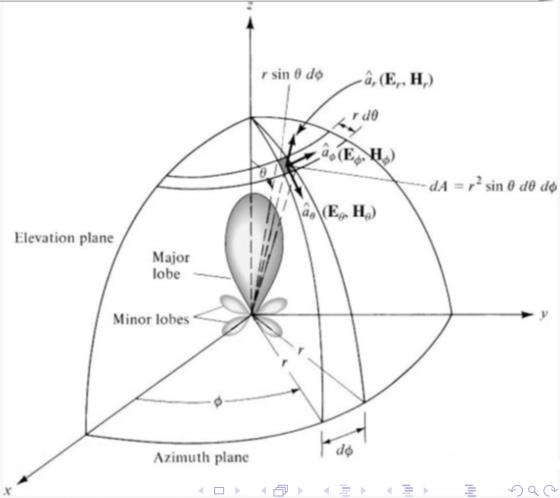
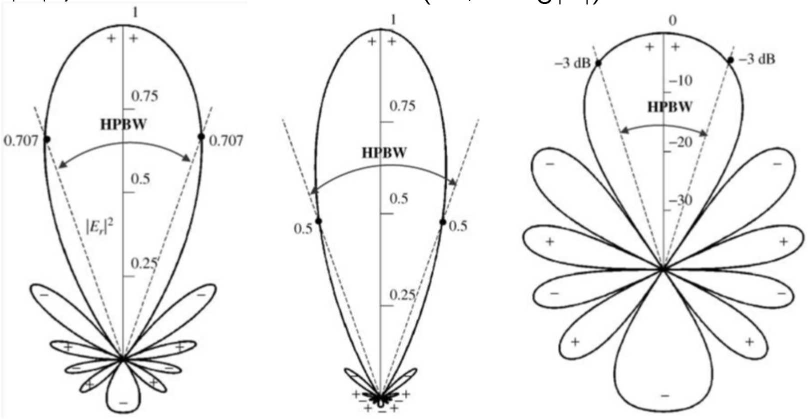


Radiation Patterns

- Field Pattern: A plot of the field magnitude ($|\mathbf{E}|$ or $|\mathbf{H}|$) on a *linear scale*.
- Power Pattern: A plot of the square of the field magnitude ($|\mathbf{E}|^2$ or $|\mathbf{H}|^2$) on either a *linear* or *decibel* (dB, $20\log|\mathbf{E}|$).

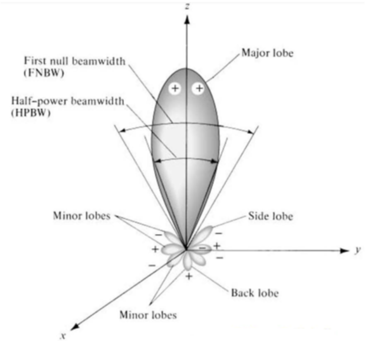


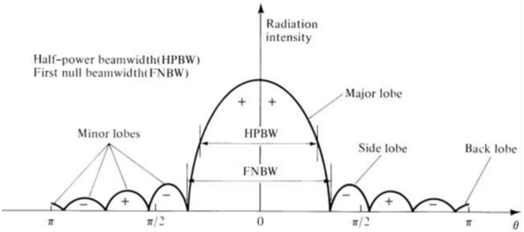
Field Pattern | Power Pattern
Linear Scale | Linear Scale | Decibel Scale (dB)

Radiation Patterns

Radiation Pattern Lobes

- Major lobe
- Minor lobes
- Side lobe
- Back lobe





Radiation Patterns

Isotropic, Directional, and Omnidirectional Pattern

Isotropic radiator

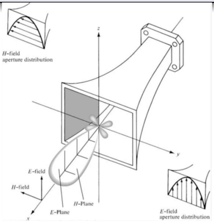
a hypothetical lossless antenna having equal radiation in all directions.

Directional antenna

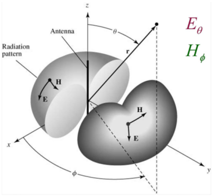
having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others, usually applied on antennas having directivity greater than that of half-wave dipole.

Omnidirectional pattern

having an essentially nondirectional pattern in a given plane.



Directional antenna



Omnidirectional in azimuth

Radiation Patterns

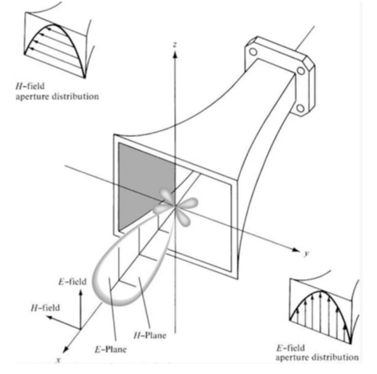
Principal Patterns

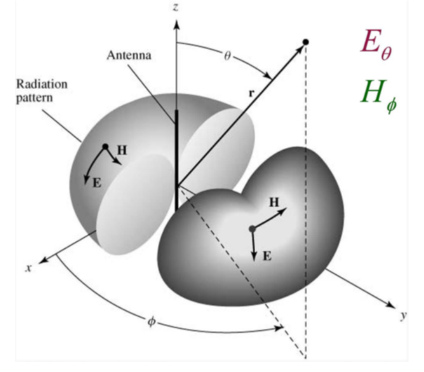
E-Plane

The plane containing the electric-field vector and the direction of maximum radiation.

H-Plane

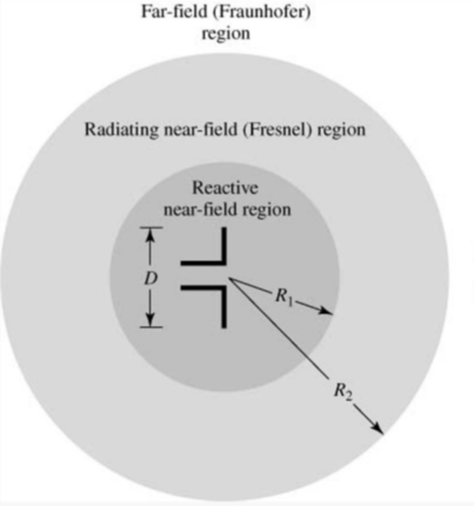
The plane containing the magnetic-field vector and the direction of maximum radiation.





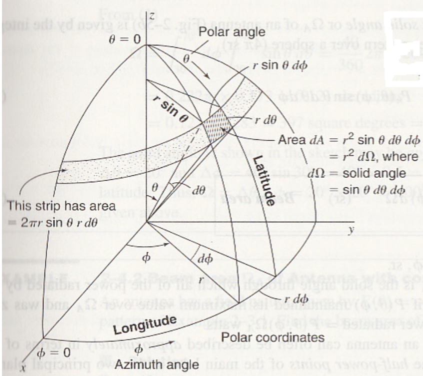
E-plane: xz plane H-plane: xy plane	E-pane: any elevation plane. H-plane: azimuth plane.

- Reactive near field region: reactive fields predominates radiating fields.
- Radiating near field (Fresnel) region: radiation fields predominates, but the radiation pattern varies with the radius r.
- Far-field (Fraunhover) region:
    - Fields vary as $\dfrac{e^{-jkr}}{r}$, and the radiation pattern is independent on r.
    - Electric and magnetic fields are predominantly in $\hat{\mathbf{a}}_\theta$ and $\hat{\mathbf{a}}_\phi$ directions, and are in phase.



Radiation Patterns

Solid Angle (Steradian)



$$dA = r^2 \sin\theta\, d\theta\, d\phi, \qquad d\Omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$$

$$\Omega = \iint_{(\theta,\phi)} \sin\theta\, d\theta\, d\phi \qquad \text{(sr)}$$

For a sphere of radius r, find the solid angle Ω_A (in square radians or steradians) of a spherical cap on the surface of the sphere over the north-pole region defined by a spherical angle of $0 \leq \theta \leq 30^\circ$, $0 \leq \phi \leq 360^\circ$.

Solution

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi/6} d\Omega = \int_0^{2\pi} \int_0^{\pi/6} \sin\theta \, d\theta \, d\phi = \int_0^{2\pi} d\phi \int_0^{\pi/6} \sin\theta \, d\theta$$

$$= 2\pi \left[-\cos\theta\right]_0^{\pi/6} = 2\pi \left[-\frac{\sqrt{3}}{2} + 1\right] = 0.83566 \quad \text{(sr)}$$

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Outline

Radiation Power Density

Instantaneous Poynting Vector

$$\mathscr{W} \text{ or } \boldsymbol{S} = \mathscr{E} \times \mathscr{H}$$

$\mathscr{W}$ or $\boldsymbol{S}$ = instantaneous Poynting vector (W/m^2).
$\mathscr{E}$ = instantaneous electric-field intensity (V/m).
$\mathscr{H}$ = instantaneous magnetic-field intensity (A/m).

$$\mathscr{P} = \oiint_S \mathscr{W} \cdot d\mathbf{s} = \oiint_S \mathscr{W} \cdot \hat{\mathbf{a}}_n da$$

$\mathscr{P}$ = instantaneous total power (W)

Radiation Power Density

$$\mathscr{E}(x,y,z;t) = \Re\left[\mathbf{E}(x,y,z)\,e^{j\omega t}\right] = \frac{1}{2}\left[\mathbf{E}e^{j\omega t} + \mathbf{E}^{*}e^{-j\omega t}\right]$$

$$\mathscr{H}(x,y,z;t) = \Re\left[\mathbf{H}(x,y,z)\,e^{j\omega t}\right] = \frac{1}{2}\left[\mathbf{H}e^{j\omega t} + \mathbf{H}^{*}e^{-j\omega t}\right]$$

$$\mathscr{W} \text{ or } \boldsymbol{S} = \mathscr{E} \times \mathscr{H} = \frac{1}{2}\Re\left[\mathbf{E} \times \mathbf{H}^{*}\right] + \frac{1}{2}\Re\left[\mathbf{E} \times \mathbf{H}e^{j2\omega t}\right]$$

Average Poynting Vector

$$\mathbf{W}_{\text{av}}(x,y,z) \text{ or } \mathbf{S}_{\text{av}}(x,y,z) = \frac{1}{2}\Re\left[\mathbf{E} \times \mathbf{H}^{*}\right]$$

$$\begin{aligned} P_{\text{rad}} &= \oiint_S \mathbf{W}_{\text{rad}} \cdot d\mathbf{s} = \oiint_S \mathbf{W}_{\text{av}} \cdot \hat{\mathbf{n}}\,da \\ &= \oiint_S \frac{1}{2}\Re\left[\mathbf{E} \times \mathbf{H}^{*}\right] \cdot d\mathbf{s} \end{aligned}$$

where P_{rad} is the average radiated power.

Radiation Power Density

Example 2.2

The radial component of the radiated power density (Poynting vector radial component) of an antenna is given by,

$$\mathbf{W}_{\text{rad}} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2),$$

Determine the total radiated power.

Radiation Power Density

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Determine the total radiated power.

Solution:

$$\begin{aligned} P_{\text{rad}} &= \oiint_S \mathbf{W}_{\text{rad}} \cdot \hat{\mathbf{n}} da \\ &= \int_0^{2\pi} \int_0^{\pi} \left(\hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \right) \cdot \left(\hat{\mathbf{a}}_r r^2 \sin\theta d\theta d\phi \right) = \pi^2 A_0 \qquad (\text{W}) \end{aligned}$$

Outline

Radiation Intensity

The power radiated from an antenna per unit solid angle,

$$U = r^2 W_{\text{rad}} \qquad \text{(W/unit solid angle)}$$

$$\begin{aligned} U(\theta,\phi) = \frac{r^2}{2\eta}\left|\mathbf{E}(r,\theta,\phi)\right|^2 &= \frac{r^2}{2\eta}\left[\left|E_\theta(r,\theta,\phi)\right|^2 + \left|E_\phi(r,\theta,\phi)\right|^2\right] \\ &= \frac{1}{2\eta}\left[\left|E_\theta^\circ(\theta,\phi)\right|^2 + \left|E_\phi^\circ(\theta,\phi)\right|^2\right] \end{aligned}$$

where far-zone electric field of the antenna,

$$\mathbf{E}(r,\theta,\phi) = \left[E_\theta^\circ(\theta,\phi)\,\hat{\mathbf{a}}_\theta + E_\phi^\circ(\theta,\phi)\,\hat{\mathbf{a}}_\phi\right]\frac{e^{-jkr}}{r}$$

Radiation Intensity

The power radiated from an antenna per unit solid angle,

$$U = r^2 W_{\text{rad}} \qquad \text{(W/unit solid angle)}$$

$$P_{\text{rad}} = \oiint_{\Omega} U d\Omega = \int_0^{2\pi} \int_0^{\pi} U \sin\theta \, d\theta \, d\phi$$

Radiation from an isotropic source

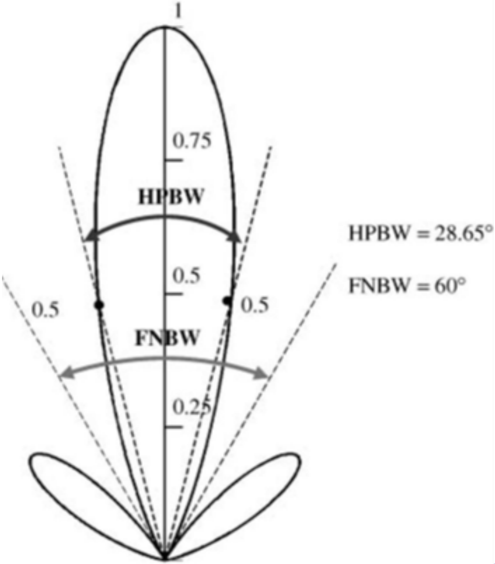
$$P_{\text{rad}} = \oiint_{\Omega} U_0 \, d\Omega = U_0 \oiint_{\Omega} d\Omega = 4\pi U_0$$

$$U_0 = \frac{P_{\text{rad}}}{4\pi}$$

Outline

Beamwidth

- HPBW (Half Power Beam Width)
- FNBW (First-Null Beam Width)



Beamwidth

Example 2.4

The normalized radiation intensity of an antenna is represented by,

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \qquad (0 \leq \theta \leq 90^\circ, \quad 0 \leq \phi \leq 360^\circ)$$

1. Find the HPBW
2. Find the FNBW

1. $U(\theta_h) = \cos^2(\theta_h)\cos^2(3\theta_h) = 0.5 \Longrightarrow \cos(\theta_h)\cos(3\theta_h) = 0.707$

   $$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right), \quad \text{iteratively gives} \quad \theta_h \approx 0.251\,\text{rad} = 14.3725^\circ$$

   $$\text{HPBW} = 2\theta_h \approx 0.502\,\text{rad} = 28.745^\circ$$

2. $U(\theta_n) = \cos^2(\theta_n)\cos^2(3\theta_n) = 0$

   $$\theta_n = \frac{\pi}{6}\,\text{rad} = 30^\circ$$

   $$\text{FNBW} = 2\theta_n = \frac{\pi}{3}\,\text{rad} = 60^\circ$$

Beamwidth

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The normalized radiation intensity of an antenna is represented by,

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1. Find the HPBW
2. Find the FNBW

1. $U(\theta_h) = \cos^2(\theta_h)\cos^2(3\theta_h) = 0.5 \implies \cos(\theta_h)\cos(3\theta_h) = 0.707$

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Outline

Directivity

Directivity

The ratio of the radiation intensity in a given direction to the radiation intensity averaged over all directions.

If the direction is not specified the direction of the maximum radiation intensity is implied.

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}}$$

$$D_{\max} = D_0 = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{\text{rad}}}$$

- Partial Directivities D_θ and D_ϕ,

$$D_\theta = \frac{4\pi U_\theta}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi}, \qquad D_\phi = \frac{4\pi U_\phi}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi}$$

$$D = D_\theta + D_\phi$$

Directivity

Example 2.6

The radial component of the radiated power density of an infinitesimal linear dipole of length $l \ll \lambda$ is given by,

$$\mathbf{W}_{\text{av}} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2}.$$

Determine the maximum directivity of the antenna and express the directivity as a function of θ and ϕ.

Directivity

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$$\mathbf{W}_{av} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2}.$$

Determine the maximum directivity of the antenna and express the directivity as a function of θ and ϕ.

Solution:

$$U = r^2 W_r = A_0 \sin^2\theta$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} A_0 \sin^2\theta \sin\theta \, d\theta \, d\phi = A_0\left(\frac{8\pi}{3}\right)$$

$$D = \frac{4\pi U}{P_{rad}} = 1.5\sin^2\theta$$

$$D_{max} = 1.5 \qquad \text{at } \theta = 90^\circ$$

Outline

Beam Solid Angle Ω_A (Beam Area)

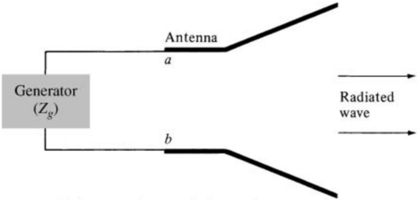
Beam Solid Angle Ω_A (Beam Area)

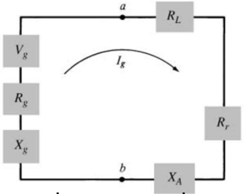
The solid angle through which all the power of the antenna would flow if its radiation is constant (and equal to the maximum value of U) for all angles within Ω_A.

$$\Omega_A = \frac{P_{\text{rad}}}{U_{max}} = \frac{\oint\!\!\oint_\Omega U d\Omega}{U_{max}} = \oint\!\!\oint_\Omega \frac{U}{U_{max}} d\Omega = \frac{4\pi}{D_{max}}$$

Outline

Antenna Input Impedance and Radiation Efficiency





- R_r = radiation resistance of the antenna
- R_L = loss resistance of the antenna
- X_A = antenna reactance
- $Z_g = R_g + jX_g$ generator impedance

Antenna Input Impedance Z_A

$$Z_A = R_A + jX_A, \qquad R_A = R_r + R_L$$

Radiation Efficiency η

The ratio of the total power radiated by an antenna to the net power accepted by the antenna from the connected transmitter.

$$\eta = \frac{P_{\text{rad}}}{P_{\text{Acc}}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{losses}}} = \frac{R_r}{R_A} = \frac{R_r}{R_r + R_L}$$

Outline

Antenna Gain

gain (in a given direction)

The ratio of the radiation intensity, in a given direction, to the radiation intensity that would be obtained if the *power accepted* by the antenna were radiated isotropically.

- Gain does not include losses arising from impedance and polarization mismatches.
- If the direction is not specified, the direction of maximum radiation intensity is implied.

$$\boxed{G = \frac{4\pi U}{P_{\text{Acc}}} = \eta D}$$

- Partial gains in θ and ϕ polarization:

$$G_\theta = \frac{4\pi U_\theta}{P_{\text{Acc}}}, \qquad G_\phi = \frac{4\pi U_\phi}{P_{\text{Acc}}}$$

$$G = G_\theta + G_\phi$$