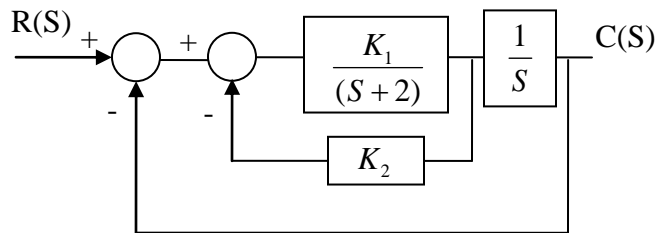


SHEET 3

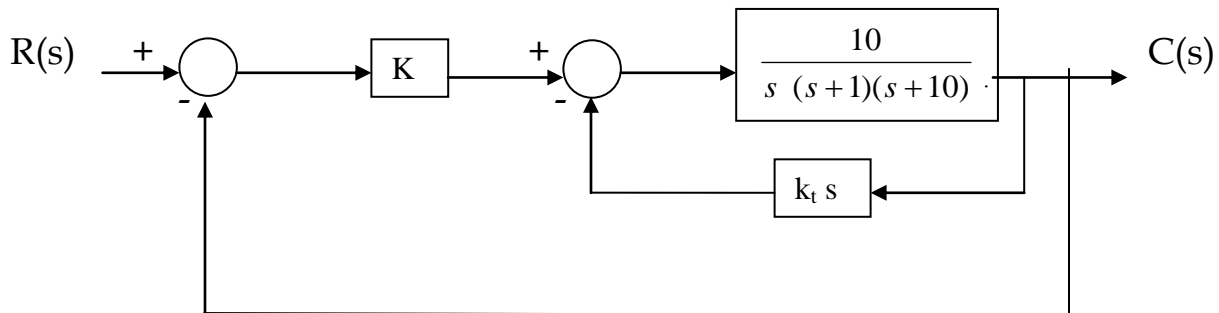
Transient Response

[1] Referring to the system shown in figure, determine the values of K_1 and K_2 such that the system has a maximum overshoot in unit step response is 25% and the peak time is 2 sec.



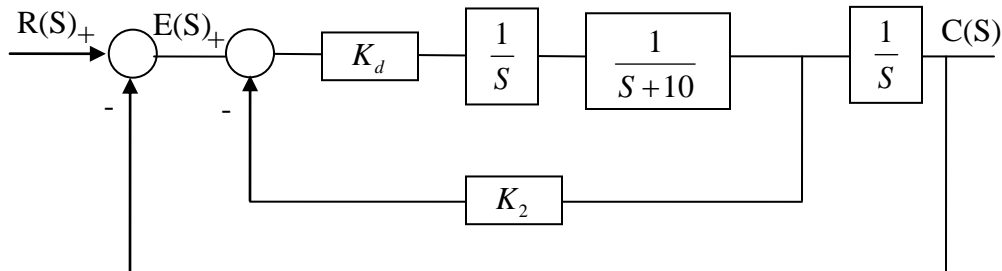
[2] The block diagram of a DC motor control system with tachometer feedback is shown in figure 4. Find the values of K and K_t so that the following specifications are satisfied:

- $k_v = 1$
- Dominant characteristic equation roots corresponding to a damping ratio of approximately 0.707



[3] (Quiz 2005) the automatic control of an airplane is one example that requires multiple-variable feedback methods. A simplified model where the rolling motion can be considered independent of other motions is shown in figure. The step response desired has an overshoot less than or equal 10% and a rise time less than or equal 4 second.

- a) Select the parameters K_d and K_2 that achieve the desired response. (8 points)
- b) Find the minimum steady state error that can be obtained by varying the values of K_d and K_2 , for a unit ramp input. (2 points)



[4](Quiz 2004) The roll control autopilot of a jet fighter is shown in figure 1 .the goal is to select a suitable K so that the response to a unit step command $\Phi_d(t)$ will provide a response $\Phi(t)$ that is a fast response and has an overshoot of less than or equal 9.5%:

- a) Using the concept of dominant poles, find a suitable value of K that will achieve the desired transient response. Predict the transient response of the system (i.e. get t_r t_p and t_s).
- b) Find the static error coefficients of the system. Evaluate the value of K that gives minimum steady state error for a unit ramp input.

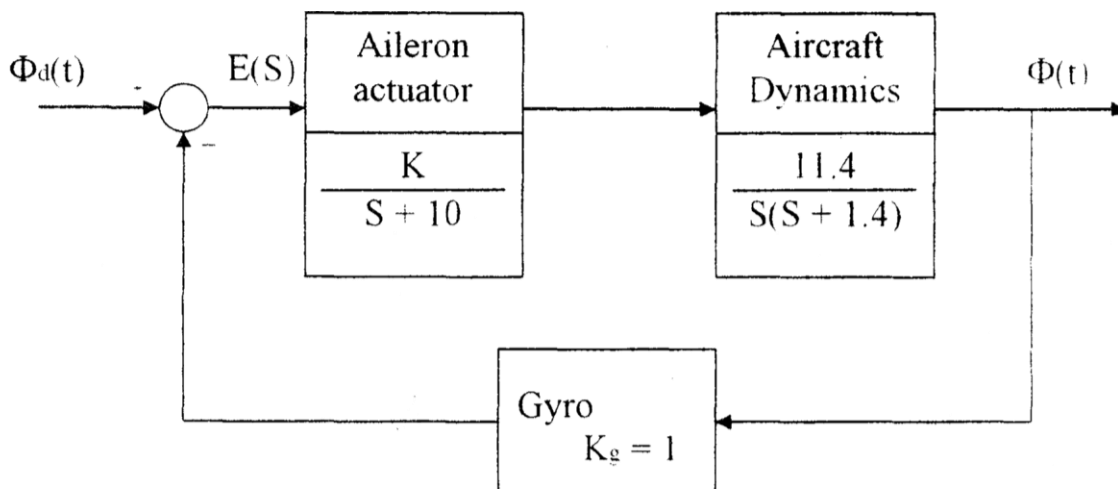
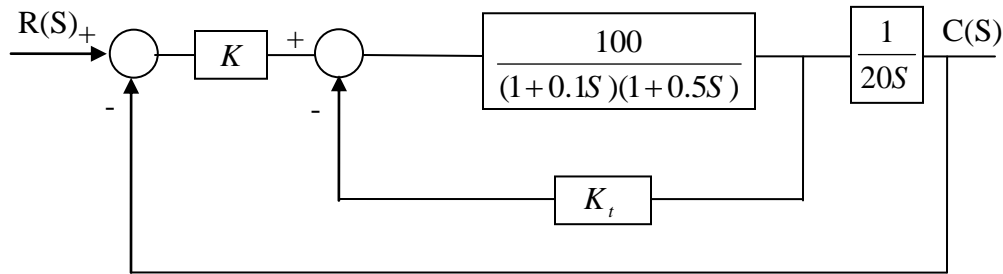


Figure (1) Roll angle control

[5](Final 2204) For the control system shown in the figure, determine the values of K and K_t such that the output has a maximum overshoot 4.3% and the rise time is approximately 2 sec. with the values of K and K_t obtained, find the steady state error when the input is a unit ramp function.



• **Summary:**

[1] For the second order continuous time system we have:

- Open loop T.F = $G(S)H(S) = \frac{\omega_n^2}{S(S + 2\zeta\omega_n)}$
- Closed loop T.F = $\frac{G(S)}{1+G(S)H(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$
- Damping ratio $\equiv \zeta$ where $0 < \zeta < 1$

$\zeta < 0.4 \Rightarrow$ excessive oscillations will occur.

$0.4 < \zeta < 0.8 \Rightarrow$ sufficiently damping & sufficiently fast.

$\zeta > 0.8 \Rightarrow$ sluggish system.

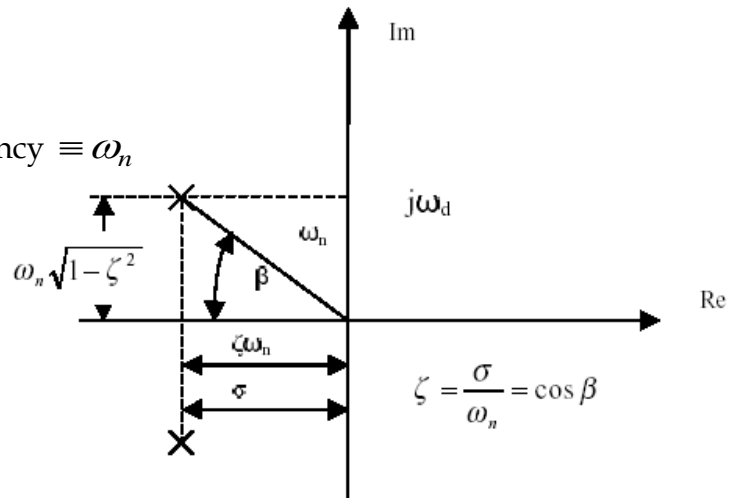
- ζ (in terms of M_p) = $\frac{-(\ln M_p)}{\sqrt{(\ln M_p)^2 + \pi^2}}$.

- Natural frequency = undamped frequency $\equiv \omega_n$

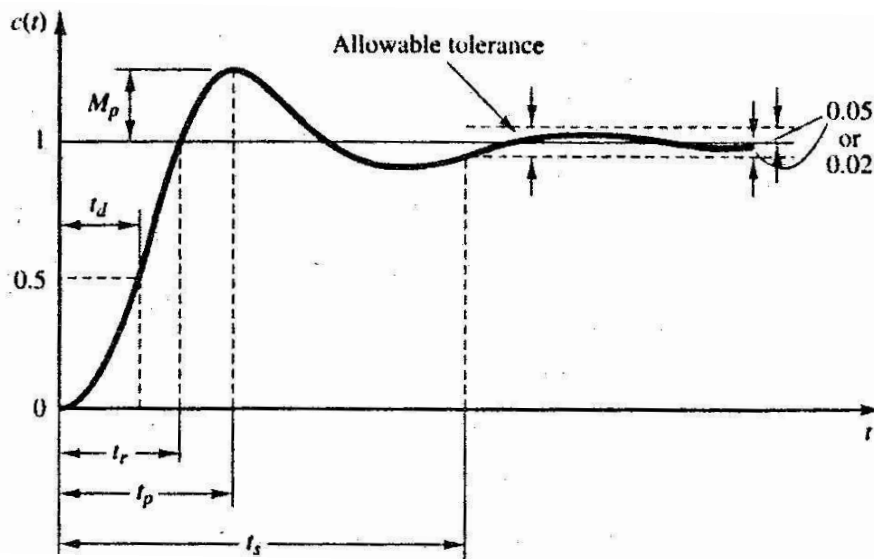
- Damped frequency $\equiv \omega = \omega_n \sqrt{1 - \zeta^2}$

- Time constant $\equiv \tau = \frac{1}{\zeta\omega_n}$

- $S_{1,2} = -\zeta\omega_n \pm j\omega = -\zeta\omega_n \pm j\omega_n \sqrt{1 - \zeta^2}$



- Rise time $\equiv t_r = \frac{\pi - \cos^{-1} \zeta}{\omega} = \frac{\pi - \cos^{-1} \zeta}{\omega_n \sqrt{1 - \zeta^2}}$
- Peak time $\equiv t_p = \frac{\pi}{\omega} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$
- Settling time $\equiv t_s = \frac{4}{\zeta \omega_n}$
- Maximum relative overshoot $\equiv M_p = e^{\frac{-\zeta \pi}{\sqrt{1 - \zeta^2}}}$



[2] For 3rd order system:

- Characteristic equation is: $(S + \alpha)(S^2 + 2\zeta\omega_n S + \omega_n^2)$.
- The dominant poles are: $(S^2 + 2\zeta\omega_n S + \omega_n^2)$, and the insignificant pole is: $(S + \alpha)$.
- The equation has three unknown: α, ζ , and ω_n .
- For design:
 1. If we have three information on the time response, we will directly get the three unknowns: α, ζ , and ω_n .
 2. If we have only two information on the time response, we need one assumption to get the three unknowns: α, ζ , and ω_n , this assumption is: $\alpha = 5\zeta\omega_n$.