Formula Sheet

- Free space parameters
  - $\epsilon_0 = \dfrac{1}{36\pi} \times 10^{-9}$ F/m
  - $\mu_0 = 4\pi \times 10^{-7}$ H/m
  - $c = 3 \times 10^8$ m/s
  - $\eta_0 = 120\pi\ \Omega$
- Plane Wave in Lossy Media, $\gamma = \alpha + j\beta$,

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \text{ Np/m}, \qquad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} \text{ rad/m}$$

- Low-Loss Dielectrics,

$$\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} \text{ Np/m}, \qquad \beta = \omega\sqrt{\mu\epsilon'}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] \text{ rad/m}, \qquad \eta_c \cong \sqrt{\frac{\mu}{\epsilon'}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right)\ \Omega$$

- Good Conductors, $\alpha = \beta = \sqrt{\pi f \mu \sigma}$, $\quad \eta_c = (1+j)\dfrac{\alpha}{\sigma}$, $\quad \delta = \dfrac{1}{\alpha}$
- Product Rules

  1. $\nabla(fg) = f\nabla g + g\nabla f$,
  2. $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$,
  3. $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$,
  4. $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$
  5. $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times\nabla f$
  6. $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

- The Three-Dimensional Delta Function

$$\delta^3(\mathbf{r}) = \delta(x)\,\delta(y)\,\delta(z), \qquad \nabla\cdot\left(\frac{\hat{\mathbf{r}}}{r^2}\right) = 4\pi\delta^3(\mathbf{r})$$

- Curvilinear Coordinates
  - Gradient: $\quad \nabla T = \dfrac{1}{h_1}\dfrac{\partial T}{\partial u_1}\hat{\mathbf{u}}_1 + \dfrac{1}{h_2}\dfrac{\partial T}{\partial u_2}\hat{\mathbf{u}}_2 + \dfrac{1}{h_3}\dfrac{\partial T}{\partial u_3}\hat{\mathbf{u}}_3$
  - Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(h_2h_3v_1) + \frac{\partial}{\partial u_2}(h_1h_3v_2) + \frac{\partial}{\partial u_3}(h_1h_2v_3)\right]$$

- Curl: $$\nabla \times \mathbf{v} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{\mathbf{u}}_1 & h_2 \hat{\mathbf{u}}_2 & h_3 \hat{\mathbf{u}}_3 \\ \partial/\partial u_1 & \partial/\partial u_2 & \partial/\partial u_3 \\ h_1 v_1 & h_2 v_2 & h_3 v_3 \end{vmatrix}$$

h_1, h_2 and h_3 are the metrics along the curvilinear coordinates u_1, u_2, and u_3, respectively.

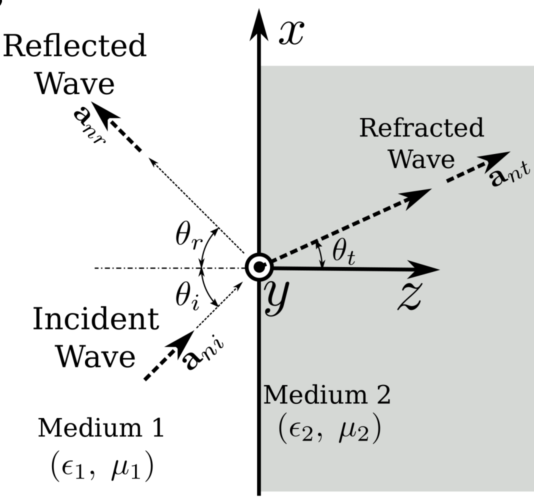
Cartesian:	$u_1 = x$, $u_2 = y$, and $u_3 = z$	$h_1 = h_2 = h_3 = 1$
Spherical Polar:	$u_1 = r$, $u_2 = \theta$, and $u_3 = \phi$	$h_1 = 1$, $h_2 = r$, $h_3 = r \sin\theta$
Cylindrical:	$u_1 = \rho$, $u_2 = \phi$, and $u_3 = z$	$h_1 = 1$, $h_2 = \rho$, $h_3 = 1$

- The infinitesimal displacement $d\mathbf{l} = h_1 du_1 \hat{\mathbf{u}}_1 + h_2 du_2 \hat{\mathbf{u}}_2 + h_3 du_3 \hat{\mathbf{u}}_3$
- Element of area $d\mathbf{a} = h_2 h_3 \hat{\mathbf{u}}_1 + h_1 h_3 \hat{\mathbf{u}}_2 + h_1 h_2 \hat{\mathbf{u}}_3$
- Element of volume $d\tau = h_1 h_2 h_3 du_1 du_2 du_3$

- Reflection Coefficient and Impedance Transformation

  Reflection coefficient Γ for incidence from medium 1 to medium 2,

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$



  - Perpendicular Polarization

$$Z_1 = \eta_1 / \cos\theta_i, \qquad Z_2 = \eta_2 / \cos\theta_t \qquad (1)$$

  - Parallel Polarization

$$Z_1 = \eta_1 \cos\theta_i, \qquad Z_2 = \eta_2 \cos\theta_t \qquad (2)$$

- Impedance Transformation

  Impedance Z_{in} at a distance d from a load impedance $Z_L$in a medium with wave impedance Z_w and z-propagation β_z is given by,

$$Z_{in} = Z_w \frac{Z_L \cos\beta_z d + j Z_w \sin\beta_z d}{Z_w \cos\beta_z d + j Z_L \sin\beta_z d}, \qquad \text{where } \beta_z = \beta \cos\theta_i$$

  where the wave impedance follows relations (1) and (2).

- Brewster Angle

  - Perpendicular polarization,

$$\sin^2\theta_{B\perp} = \frac{1 - \dfrac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}}{1 - (\mu_1/u_2)^2}$$

  - Parallel polarization,

$$\sin^2\theta_{B\|} = \frac{1 - \dfrac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2}}{1 - (\epsilon_1/\epsilon_2)^2}$$