2CAIRO UNIVERSITY
ELECTRONICS & COMMUNICATIONS DEP.
CONTROL ENGINEERING

FACULTY OF ENGINEERING
3rd YEAR, 2010/2011

SHEET 2

Bode Plot

[1] Draw the asymptotic Bode plots for each of the following transfer functions:

a) $G(s) = \dfrac{2(1+0.5s)}{(1+0.1s)(1+0.4s)}$

b) $G(s) = \dfrac{2(1+0.4s)}{s^2(1+0.1s)(1+0.05s)}$

[2] A servomotor with field control has a mechanical time constant of 0.1 sec., a field electrical time constant of 0.008 sec., and a velocity gain constant of 200 rpm/v. Sketch the Bode plots for the velocity transfer function $\dfrac{\theta(s)}{v_f(s)}$ of the motor.

[3] A unity feedback system has $G(s) = \dfrac{1}{s(s+1)(s+0.2)}$:

a) Draw the asymptotic Bode plots for the above system.
b) Determine the gain crossover and the phase crossover frequencies.
c) Determine the gain and phase margins
d) Is the closed loop system stable? Check using Routh method.

[4] (Midterm 2003) a unity feedback control system has $G(s) = \dfrac{1}{2S(S+1)^2}$:

a) Construct the asymptotic Bode plots.
b) From the plots, determine the gain K_1 required to have a critically stable system.
c) From the plots, find K_2 for a GM = 10 db, then get PM_{new}.
d) Find the pure delay required to have a PM=20^o.

[5](Midterm 2004)A unity feedback control system has $G(s)=\dfrac{9}{(S+0.5)(S^2+3S+9)}$:

(a) Draw the asymptotic bode plots for the above system.

(b) Suppose that a gain K_1 is cascaded with G(S).Find the value of K_1 to have a GM of 20 db, then get the new PM, ω_{gc} *and* ω_{pc}.

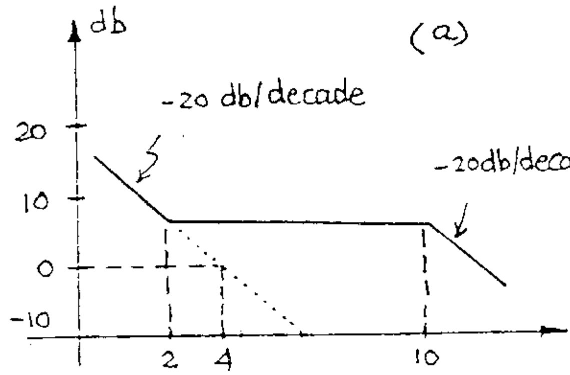
(c) Suppose that $K_2 e^{-\alpha s}$ is cascaded with G(S) (instead of K_1).Determine the value of K_2 and α to make the system critically stable with $\omega_{gc}=2$ rad/sec.

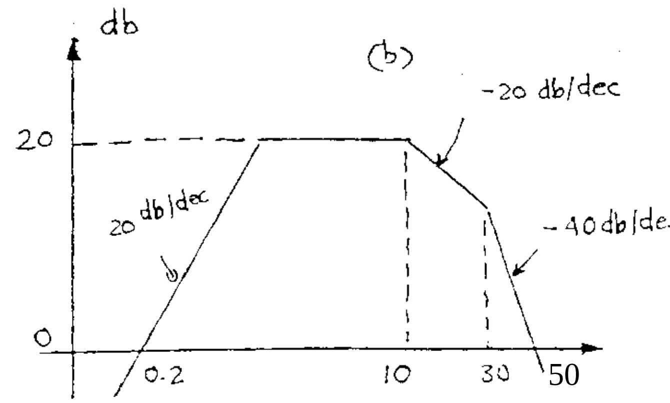
[6](Final 99)

a) Determine the transfer function corresponding to the Bode plots shown below assuming that they are minimum phase.

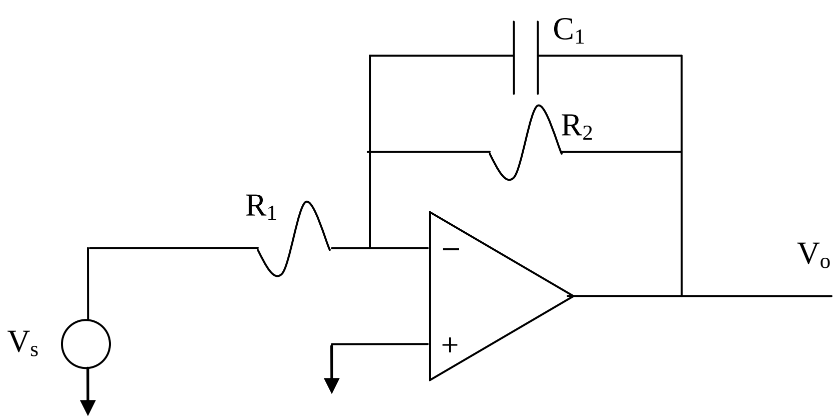
b) Discuss the stability of each system.

c) If the system in fig (b) is used in closed unity feedback loop system and the controller is a pure time delay $e^{-\alpha s}$, calculate the delay time α for a PM=30°.





[7] Consider the circuit shown:



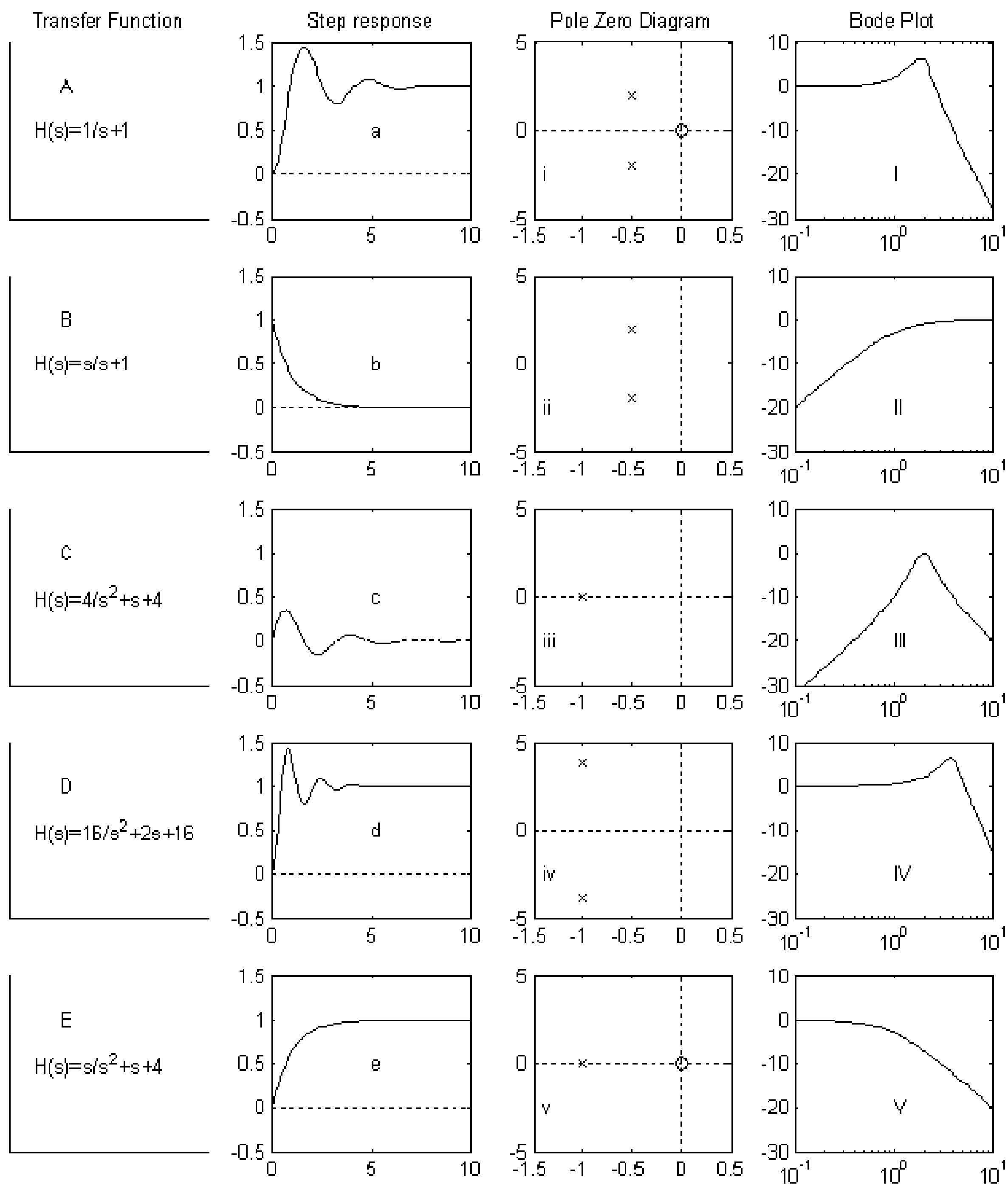
a) Analyze the circuit to get the transfer function $H(S)=\dfrac{V_o(S)}{V_s(S)}$.

b) If the time constant of the system equals 0.1 msec and the DC gain equals -10, draw the Bode plot of the system.

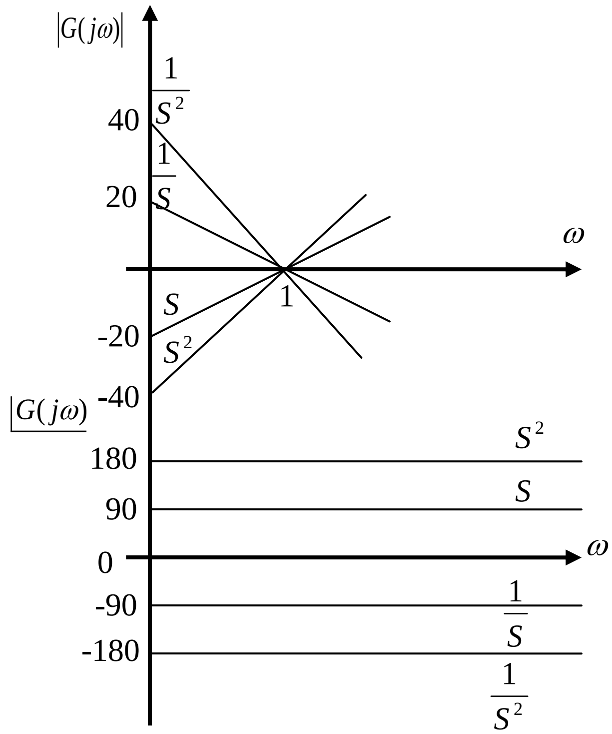
c) Form the plots, what is the function of this circuit?

[8] The image below shows several transfer functions. For each transfer function, there is also a single matching Pole-Zero diagrams, a Bode Plot, and a Step Response. Your job is to match them up by filling out the table (e.g., which step response goes with Transfer function "A"?).

Transfer Function	Step Response	Pole Zero Diagram	Bode Plot
A			
B			
C			
D			
E			



- ***Summary of Bode Plot:***



1) *Poles or zeros at origin of order n:*

$$G(S)=S^{\pm n}=(j\omega)^{\pm n}$$

$$|G(S)|=\pm 20n\log\omega,\ \underline{|G(S)}=\pm n*90^\circ$$

The slope of the mag. Line = $\pm 20n\ db/decade$

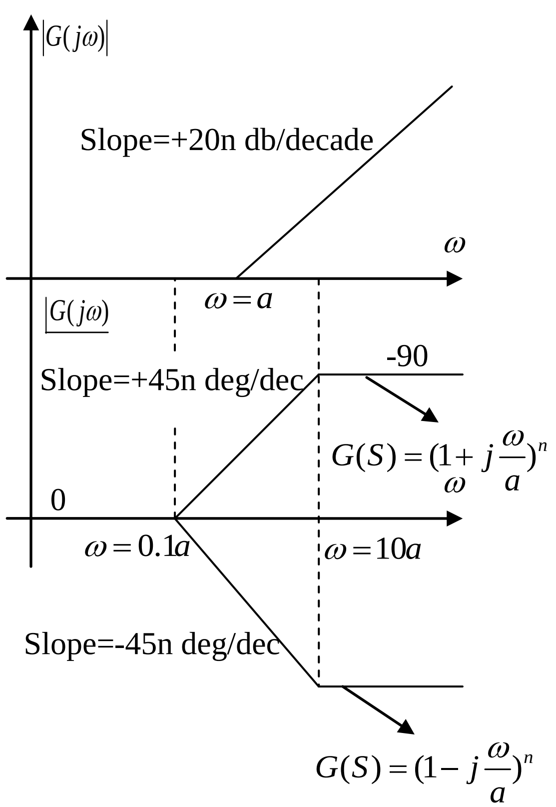
These lines pass through w=1

2) Simple Pole or Zero of order n:

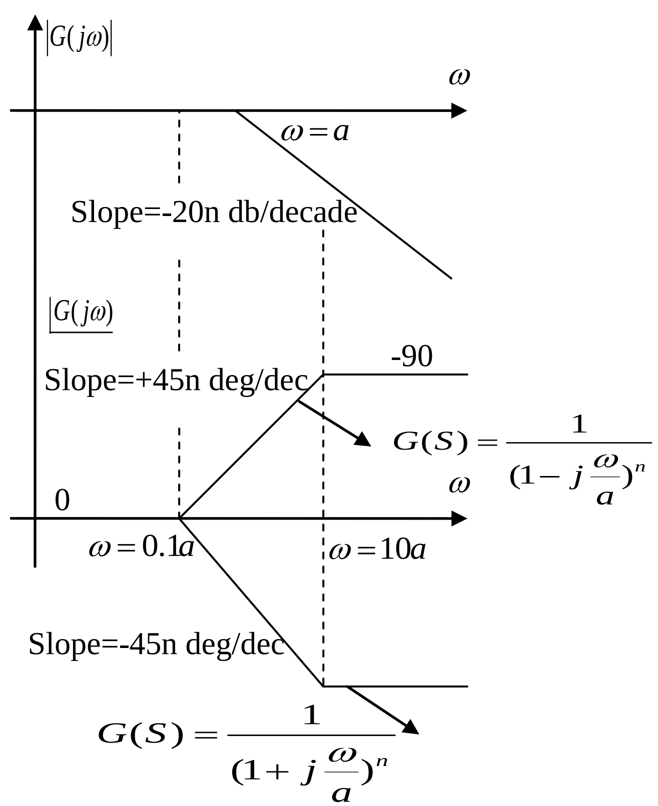
$$G(S)=(1\pm\frac{S}{a})^{\pm n}=(1\pm j\frac{\omega}{a})^{\pm n}$$

$$|G(S)|=\left\{\sqrt{\left(1+\left(\frac{\omega}{a}\right)^2\right.}\right\}^{\pm n},\ \underline{|G(S)}=(\pm n)\pm\tan^{-1}\frac{\omega}{a}$$

$$G(S)=(1\pm j\frac{\omega}{a})^n$$



$$G(S)=\frac{1}{(1\pm j\frac{\omega}{a})^n}$$



3) Quadratic poles and zeros: $G(S) = \dfrac{\omega_n^{\,2}}{S^2 + 2\xi\omega_n S + \omega_n^{\,2}}$

- For magnitude:

1) Determine the natural frequency from G(s).

2) ***Before*** ω_n no change in the magnitude, ***After*** ω_n there is a change equals -40 db/decade.

- For phase:

1) $\angle G(S) = -\tan^{-1}\dfrac{2\xi\omega_n}{\omega_n^{\,2} - \omega^2}$ Calculate the phase at 0.1 ω_n, ω_n, and 10 ω_n.

2) The phase at ω =0 equals zero, at $\omega = \omega_n$ equals -90, at $\omega = \infty$ equals -180.

4) Pure time Delay: $e^{-j\omega T_d}$

- No effect on magnitude
- Its phase = $-\omega T_d * \dfrac{180}{\pi}$,which means that any pure delay in the system will ***shift down*** the phase plot of the original system by $\omega T_d * \dfrac{180}{\pi}$.

5) Relation between system type and bode plot:

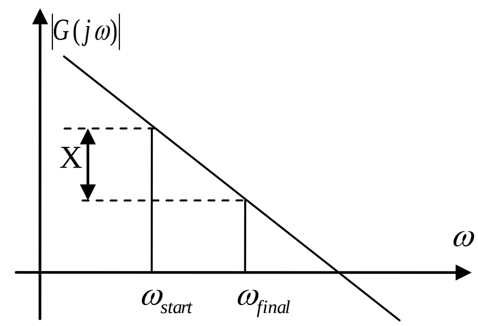
- For type 0 system: We begin the mag. Plot with a horizontal line at 20 log K_p.
- For type 1 system: We begin the mag. Plot with a line of slope -20db/dec, and this line intersects with the 0 db line at K_v.
- For type 2 system: We begin the mag. Plot with a line of slope -40db/dec, and this line intersects with the 0 db line at $\sqrt{K_a}$.

6) Definitions:

- gain crossover frequency : ω_{gc}, $|G(j\omega_{gc})| = 1 \Leftrightarrow |G(j\omega_{gc})|_{db} = 20\log|G(j\omega_{gc})| = 0$ (from graph)
- phase crossover frequency : ω_{pc} , $\angle G(j\omega_{pc}) = -180$ (from graph)
- Gain margin: $GM = -|G(j\omega_{pc})|_{db}$ (from graph)
- Phase margin: $PM = 180 + \angle G(j\omega_{gc})$ (from graph)
- The system is stable iff: (GM and PM are both +ve) $\Leftrightarrow (\omega_{gc} < \omega_{pc})$

7) Plotting Rules:

a) For Magnitude:

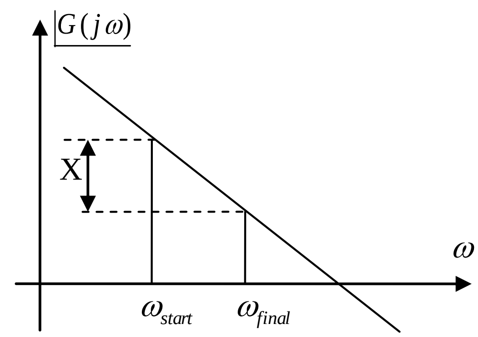


- Determine **the corner frequencies** on the plot.

- Draw the magnitude using the following rule:

$$|X| = |slope| \log \frac{\omega_{final}}{\omega_{start}}$$

b) For Phase:



- Determine the **points lies before and after decade for Each corner frequency**.
- Draw the phase using the following rule:

$$|X| = |slope| \log \frac{\omega_{final}}{\omega_{start}}$$