Formula Sheet

Free space parameters

- $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m
- $\mu_0 = 4\pi \times 10^{-7}$ H/m
- $c = 3 \times 10^8$ m/s
- $\eta_0 = 120\pi\ \Omega$

Plane Wave in Lossy Media, $\gamma = \alpha + j\beta$,

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2} \text{ Np/m}, \qquad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2} \text{ rad/m}$$

Low-Loss Dielectrics,

$$\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} \text{ Np/m}, \qquad \beta = \omega\sqrt{\mu\epsilon'}\left[1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] \text{ rad/m}, \qquad \eta_c \cong \sqrt{\frac{\mu}{\epsilon'}}\left(1+j\frac{\epsilon''}{2\epsilon'}\right)\ \Omega$$

Good Conductors,

$$\alpha = \beta = \sqrt{\pi f\mu\sigma}, \qquad \eta_c = (1+j)\,\frac{\alpha}{\sigma}, \qquad \delta_s = \frac{1}{\alpha}$$

Product Rules

1. $\nabla(fg) = f\nabla g + g\nabla f$,
2. $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$,
3. $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$,
4. $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$
5. $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times\nabla f$
6. $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

The Three-Dimensional Delta Function

$$\delta^3(\mathbf{r}) = \delta(x)\,\delta(y)\,\delta(z)$$
$$\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right) = 4\pi\delta^3(\mathbf{r})$$

Curvilinear Coordinates

- Gradient: $\nabla T = \frac{1}{h_1}\frac{\partial T}{\partial u_1}\hat{\mathbf{u}}_1 + \frac{1}{h_2}\frac{\partial T}{\partial u_2}\hat{\mathbf{u}}_2 + \frac{1}{h_3}\frac{\partial T}{\partial u_3}\hat{\mathbf{u}}_3$
- Divergence:

$$\nabla \cdot \mathbf{v} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 v_1) + \frac{\partial}{\partial u_2}(h_1 h_3 v_2) + \frac{\partial}{\partial u_3}(h_1 h_2 v_3)\right]$$

- Curl: $\nabla \times \mathbf{v} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{\mathbf{u}}_1 & h_2\hat{\mathbf{u}}_2 & h_3\hat{\mathbf{u}}_3 \\ \partial/\partial u_1 & \partial/\partial u_2 & \partial/\partial u_3 \\ h_1 v_1 & h_2 v_2 & h_3 v_3 \end{vmatrix}$

  h_1, h_2 and h_3 are the metrics along the curvilinear coordinates u_1, u_2, and u_3, respectively.

Cartesian:	$u_1 = x$, $u_2 = y$, and $u_3 = z$	$h_1 = h_2 = h_3 = 1$
Spherical Polar:	$u_1 = r$, $u_2 = \theta$, and $u_3 = \phi$	$h_1 = 1$, $h_2 = r$, $h_3 = r\sin\theta$
Cylindrical:	$u_1 = \rho$, $u_2 = \phi$, and $u_3 = z$	$h_1 = 1$, $h_2 = \rho$, $h_3 = 1$

- The infinitesimal displacement $d\mathbf{l} = h_1 du_1\hat{\mathbf{u}}_1 + h_2 du_2\hat{\mathbf{u}}_2 + h_3 du_3\hat{\mathbf{u}}_3$
- Element of area $d\mathbf{a} = h_2 h_3\hat{\mathbf{u}}_1 + h_1 h_3\hat{\mathbf{u}}_2 + h_1 h_2\hat{\mathbf{u}}_3$
- Element of volume $d\tau = h_1 h_2 h_3 du_1 du_2 du_3$

Potentials

- **Curl-less** (or "**irrotational**") fields. The following conditions are equivalent (that is, $\mathbf{F}$ satisfies one if and only if it satisfies all the others):

  $\nabla \times \mathbf{F} = \mathbf{0}$ everywhere.

  1. $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F} \cdot d\mathbf{l}$ is independent of path, for any given end points.
  2. $\oint \mathbf{F} \cdot d\mathbf{l} = 0$ for any closed loop.
  3. $\mathbf{F}$ is the gradient of some scalar, $\mathbf{F} = -\nabla V$.

- **Divergence-less** (or "**solenoidal**") fields. The following conditions are equivalent:

  $\nabla \cdot \mathbf{F} = \mathbf{0}$ everywhere.

  1. $\int_S \mathbf{F} \cdot d\mathbf{a}$ is independent of surface, for any given boundary line.
  2. $\oint \mathbf{F} \cdot d\mathbf{a} = 0$ for any closed surface.
  3. $\mathbf{F}$ is the curl of some vector, $\mathbf{F} = \nabla \times \mathbf{A}$.

- Any vector $\mathbf{F}$ can be written as $\mathbf{F} = -\nabla V + \nabla \times \mathbf{A}$