CAIRO UNIVERSITY
ELECTRONICS & COMMUNICATIONS DEP.
CONTROL ENGINEERING

FACULTY OF ENGINEERING
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SHEET 1

Error Analysis

1. A unity feedback system has a forward transfer function of:

$$G(s) = \frac{12\,(s+4)}{s\,(s+1)\,(s+3)\,(s^2+2s+10)}$$

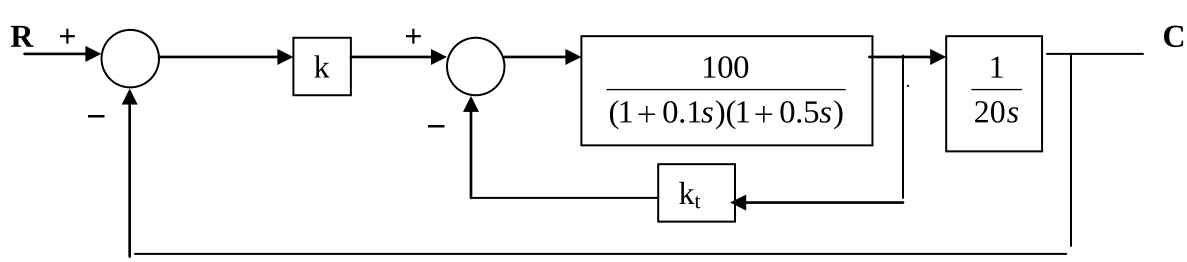
a) Determine the static error coefficients for this system.
b) Determine the steady state error and the steady state output for a reference input r (t) =16+2t and for r (t) =5t^2.
c) Is the closed loop system stable?

2. A unity feedback control system has the forward transfer function:

$$G(s) = \frac{k_v}{s\,(4s+1)\,(s+1)}$$

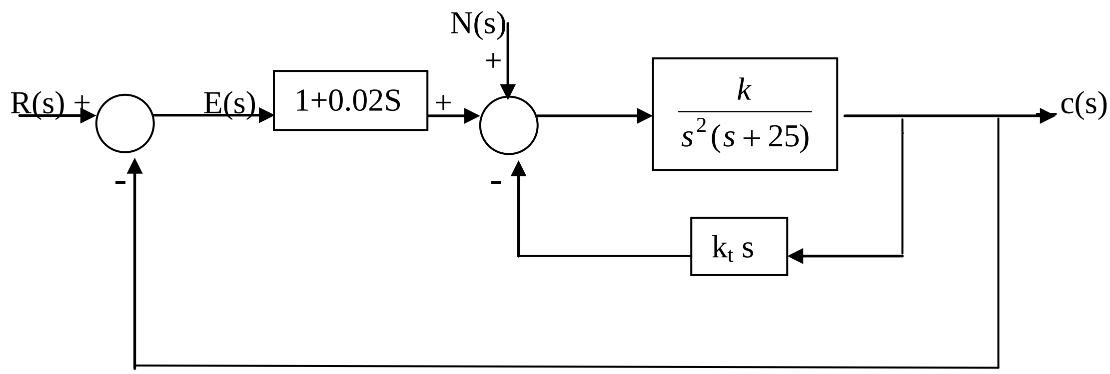
a) The steady state value of the error is desired to be less than or equal to 0.1 for a reference input r (t) = 1+t. Determine the minimum value of k_v that satisfies this requirement.
b) Check the stability of the system for the value k_v of obtained in part (a) and comment on your result.

3. The block diagram of a control system is shown the following figure. Find the step, ramp and parabolic error constants. The error signal is defined to be e (t). find the steady state errors of the system in terms of k and k_t, when the following inputs are applied:
a) r (t) =6+8t
b) r(t) =2t+7 t^2
What constraint must be made on the values of k and k_t, so that the answers are valid? Determine the minimum steady state error that can be achieved with a unit ramp input.



4. The block diagram of a feedback control system is shown in the figure. The error signal is defined to be e (t).

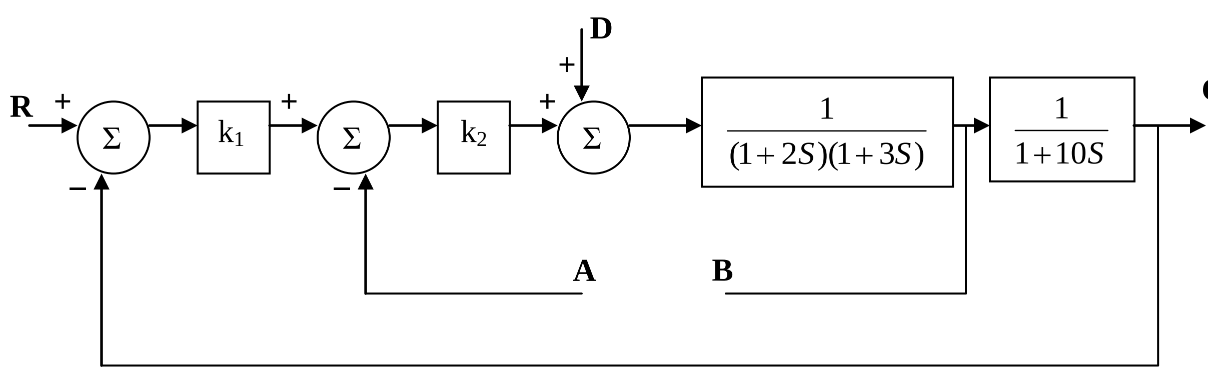
a) Find the steady state error of the system in terms of k and k_t when the input is a unit ramp function. Give the constraints on the values of k and k_t so that the answer is valid. Let n (t) =0 for this part.
b) Find the steady state value of c (t) when n (t) is a unit step function. Let r (t) =0



5. (Final 98) The following Figure shows a block diagram of a cascade control system designed to counteract disturbance D.
(a) Initially consider the situation with no cascade loop, i.e. link AB open.

- State the type of the system and find the static error coefficients in terms of loop gains.
- Determine the limiting value of the gain (K_1K_2) in order to maintain stable operation.
- Let K_2 = 50, Find the value of K_1 that gives steady state output due a unit step disturbance equal (0.1).

Link AB is closed to implement the cascade loop and K_1, K_2 are fixed at the values obtained in part (a). Find the steady state change in output due to a unit step disturbance.



6. A control system with unity feedback has the forward transfer function:

$$G(s)\frac{k}{(s+1)^2(s+4)}$$

a) Evaluate the error series when the input r (t) = 6+3t is applied, hence find the value of the steady state error at t=2 sec. in terms of k.
b) Find the minimum value of the steady state error that can be achieved by varying the value of k for a unit step input.

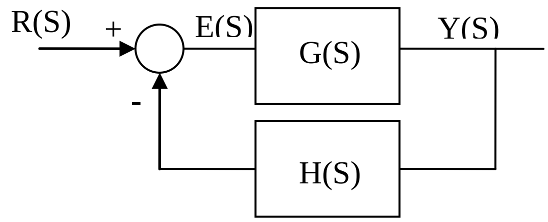
7. (Final 2001) A control system with unity feedback has the transfer function:

$$G(S)=\frac{K}{(S+1)(S^2+2S+2)}$$

(a) Evaluate the error series when the input r (t) = 10 + 6t + e^{-5t} is applied.
(b) Find the value of the error (in terms of K) at t = 3 sec.
(c) Find the minimum value of steady state error that can be achieved by varying the value of K for a unit step input.

- ***Summary:***

A. For the continuous time system:

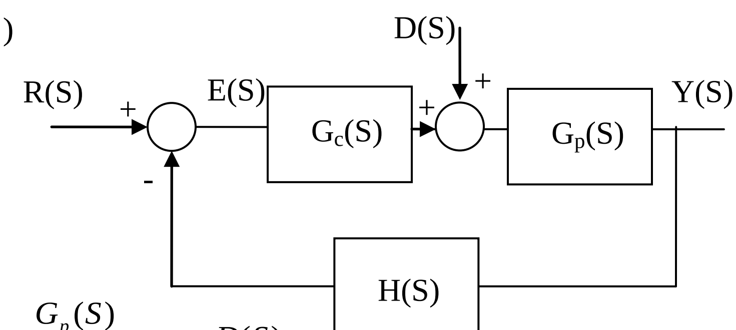


- $E(S) = \dfrac{R(S)}{1 + G(S)H(S)}$
- $E(S)$ □ exact expression of the error
- $e_{ss} = \lim_{s \to 0} SE(S) = \lim_{t \to \infty} e(t)$ ***or*** you can easily get it from the following table:

	R(S)	e_{ss}	Error const. value
e_{ss} due to step input	$\dfrac{M}{S}$	$\dfrac{M}{1+K_p}$	$K_p = \lim_{s \to 0} G(S)H(S) \equiv$ position error constant
e_{ss} due to ramp input	$\dfrac{M}{S^2}$	$\dfrac{M}{K_v}$	$K_v = \lim_{s \to 0} SG(S)H(S) \equiv$ velocity error constant
e_{ss} due to parab input	$\dfrac{M}{S^3}$	$\dfrac{M}{K_a}$	$K_a = \lim_{s \to 0} S^2 G(S)H(S) \equiv$ acceleration error constant

- $Y_{ss}(S) = \lim_{s \to 0} SY(S) = R_{ss}(S) - e_{ss}$

B. For a continuous time system with a disturbance:



- $Y(S)\Big|_{\text{due to disturbance D(S)}} = \dfrac{G_p(S)}{1 + G_p(S)G_c(S)H(S)} D(S)$
- $e_{ss}\Big|_{\text{due to dist.}} = -\lim_{s \to 0} SY(S)\Big|_{\text{due to disturbance D(S)}}$
- $Y(S)\Big|_{\text{due to input R(S)}} = \dfrac{G_p(S)G_c(S)}{1 + G_p(S)G_c(S)H(S)} R(S)$
- $Y(S)\Big|_{\text{Total}} = \dfrac{G_p(S)G_c(S)}{1 + G_p(S)G_c(S)H(S)} R(S) + \dfrac{G_p(S)}{1 + G_p(S)G_c(S)H(S)} D(S)$

C. Error series:

- $$e_s(t) = C_0 r(t) + C_1 \overset{o}{r}(t) + C_2 \frac{\overset{oo}{r}(t)}{2}$$

  where :

  $$C_0 = \lim_{s \to 0} F(S)\ ,\ C_1 = \lim_{s \to 0} \frac{dF(S)}{dS}\ ,\ C_2 = \lim_{s \to 0} \frac{d^2F(S)}{dS^2}$$

  $$\text{and } \ F(S) = \frac{E(S)}{R(S)} = \frac{1}{1 + G(S)H(S)}$$