Formula Sheet

- Radiation electric field from dipole of length l along z-axis,

$$\mathbf{E} = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right] \hat{\mathbf{a}}_\theta$$

- For infinitesimal dipole of length l ($l \ll \lambda$) along direction $\hat{\mathbf{n}}$,
  - Radiation electric field

$$\mathbf{E} = -j\eta \frac{klI_0 e^{-jkr}}{4\pi r} \left[\hat{\mathbf{n}} - (\hat{\mathbf{n}} \cdot \hat{\mathbf{a}}_r)\, \hat{\mathbf{a}}_r\right]$$

  - Radiation Resistance

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2.$$

- Small Loop Antenna
  - Radiation electric field from a small loop of radius a ($2\pi a < \lambda/10$) with its axis oriented along direction $\hat{\mathbf{n}}$,

$$\mathbf{E} = \eta \frac{k^2 a^2 I_0 e^{-jkr}}{4r} \hat{\mathbf{n}} \times \hat{\mathbf{a}}_r$$

  - Radiation resistance of N-turns small loop antenna of radius a

$$R_r = 20\pi^2 \left(\frac{2\pi a}{\lambda}\right)^4 N^2 \quad \Omega$$

  - Ohmic loss resistance N-turns small loop antenna of radius a and wire radius b and surface resistance R_s,

$$R_l = \frac{Na}{b} R_s$$

- Directivity of $\lambda/2$ dipole is 1.643 and radiation resistance is 73 Ω.

- Conductor surface resistance $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}}$, where $\mu_0 = 4\pi \times 10^{-7}$ H/m.

- Matrix transformation of any vector from Cartesian to spherical coordinates,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

- $$\int e^{\alpha x}\sin(\beta x+\gamma) = \frac{e^{\alpha x}}{\alpha^2+\beta^2}\left[\alpha\sin(\beta x+\gamma) - \beta\cos(\beta x+\gamma)\right]$$

- Angle ψ between the two directions (θ, ϕ) and (θ', ϕ'),

$$\cos\psi = \sin\theta\sin\theta'\cos(\phi-\phi') + \cos\theta\cos\theta'$$

- Beam solid angle Ω_A,

$$\Omega_A = \frac{\oiint_{4\pi} U d\Omega}{U_{max}}$$

- Aperture Efficiency ϵ_{ap} is,

$$\epsilon_{ap} = \frac{\left|\iint_{A_p} \mathbf{E}_a ds'\right|^2}{A_p \iint_{A_p} \left|\mathbf{E}_a\right|^2 ds'}$$

- Antenna Vector Effective Length $\boldsymbol{\ell}$,

$$\mathbf{E} = j\eta\frac{kI_{in}}{4\pi r}\boldsymbol{\ell}e^{-jkr}$$

- Radiation fields due to electric and magnetic surface currents,

$$E_\theta = -j\omega\left(A_\theta + \eta F_\phi\right), \qquad E_\phi = -j\omega\left(A_\phi - \eta F_\theta\right)$$

$$\mathbf{A} = \frac{\mu e^{-jkr}}{4\pi r}\mathbf{N}, \qquad \mathbf{F} = \frac{\epsilon e^{-jkr}}{4\pi r}\mathbf{L}$$

$$\mathbf{N} = \iint_S \mathbf{J}_s e^{jkr'\cos\psi} ds' = \iint_S \mathbf{J}_s e^{j\mathbf{k}\cdot\mathbf{r}'} ds', \qquad \mathbf{L} = \iint_S \mathbf{M}_s e^{jkr'\cos\psi} ds' = \iint_S \mathbf{M}_s e^{j\mathbf{k}\cdot\mathbf{r}'} ds',$$

where $\mathbf{k}\cdot\mathbf{r}' = k\left(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta\right)$

Electric current on a perfect electric conductor: $\mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}$

Magnetic current on a perfect magnetic conductor: $\mathbf{M}_s = -\hat{\mathbf{n}} \times \mathbf{E}$