# CHAPTER 9

## VELOCITY MODULATION AND KLYSTRON BUNCHING

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**9·1. Introduction.**—It is pointed out in Chap. 1 that the conventional multielectrode tube encounters serious limitations at microwave frequencies, and that some of these limitations may be minimized by new techniques of vacuum-tube construction. Regardless of circuit improvements thus made possible, however, there remains the basic electronic necessity for transit of the electrons through the control (i.e., cathode-grid) region in a time considerably smaller than a cycle of the microwave oscillation in question. Since it is an essential feature of such tubes that the electron velocity in the cathode-grid region never exceeds a value corresponding to a small fraction of the plate voltage, the requirement of short transit time becomes a very stringent requirement on interelectrode spacing.

The basic electronic problem in these tubes and in any oscillator or amplifier is, in general, the problem of utilizing an r-f voltage (derived from feedback or input) to produce at some other point a conduction current with an r-f component—that is, it is the problem of producing an electronic transfer admittance, or transadmittance.

The klystron[1] is the product of an approach to the transadmittance problem that differs radically from previously described (and historically antecedent) approaches, and was stimulated by the difficulties encountered in the latter. Electronically there are two marked innovations in the klystron. The most important of these is the combined process of velocity modulation and bunching, by which the finite transit time of electrons becomes the basic means of producing (rather than a limitation upon) the transadmittance. This process of velocity modulation and bunching is the element common to all klystrons, and it is therefore discussed in some detail in this chapter before the various types of klystron are described.

The second radical departure in the klystron is made possible by the first and is not discussed further in itself—it is the application of the velocity-modulating r-f control voltages to the electrons after, rather than before, the acceleration of the electrons by the full applied plate voltage. Although the electrons must preferably have a transit time

[1] R. H. Varian and S. F. Varian, *Jour. App. Phys.*, **10,** 321 (1939).

through the control region of less than one cycle, the electron velocity is much higher for a given plate voltage and the geometrical limitations on the control region are therefore greatly relaxed.

The present chapter is intended to be a reference compendium of the basic information about velocity modulation and bunching that will be required for subsequent discussion of the various forms of klystrons. Thus, the choice of material has been governed primarily by the topics covered in the later chapters rather than by any desire to summarize completely all the features of bunching that would be necessary to form a complete discussion of this very interesting field. For the same reason, all discussion of the way in which klystron behavior is affected by the details of the bunching process is left for the later chapters. It is therefore suggested that the reader may profitably confine a first reading to Section 9·2 of the present chapter, returning to the other sections as they are referred to in later chapters.

**9·2. Simple Velocity Modulation and Bunching.**—The schematic diagram in Fig. 9·1 represents, in an idealized form, that part of a klystron in which the processes of velocity modulation and bunching take place. This region corresponds to the input or cathode-grid space of a triode, in the sense that from this region there emerges an intensity-modulated conduction current that serves to drive the output cavity resonator. The nature of microwave cavity resonators and the way in which they are driven by an r-f component of the conduction current is discussed in Chaps. 3 and 4; this chapter is concerned only with the genesis of the electronic transadmittance to which this r-f conduction current corresponds.

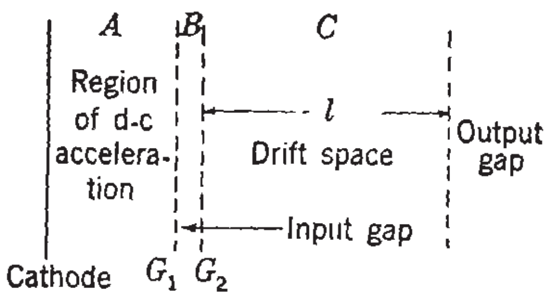


FIG. 9·1.—Schematic representation of velocity modulation and bunching region in klystron.

The space shown in Fig. 9·1 comprises three separate regions. The processes that occur in these regions are first qualitatively summarized, temporarily making simplifications in order to emphasize the fundamental points.

In region $A$—the space between the cathode $K$ and the grid $G_1$—there exists only a d-c field that corresponds to the application of full beam voltage between $K$ and $G_1$. The influence of the d-c field results in the injection into region $B$, through $G_1$, of a stream of electrons all having the same velocity $v_0$ (given by $mv_0^2/2 = eV_0$) and with current density constant in time.

In region $B$—the control region (or "input gap") between grids $G_1$ and $G_2$—there is an externally impressed alternating r-f voltage the instantaneous value of which is written as $V \sin \omega t$. This instantaneous voltage is the real part of the complex voltage $V_1e^{j\omega t}$; the complex voltage amplitude $V_1$ is thus given by

$$V_1 = -jV. \tag{1}$$

Throughout the present section it is assumed, for simplicity, that $V/2V_0 \ll 1$, and that the time of transit through region $B$ is very small compared with one cycle of the r-f oscillation. (This transit-time condition is easier to meet here than in the analogous cathode-grid region of a triode because the electrons have already received full d-c acceleration in region $A$.) No electrons are turned back between $G_1$ and $G_2$, and the current density of the stream of electrons leaving $G_2$ is closely constant in time, just as it was at $G_1$. Individual electrons are speeded up or slowed down in passage through the input gap, depending on the phase of the r-f field at the time of the electron's transit. Adopting the convention that the r-f voltage is positive when electrons are accelerated, it follows that each electron in passage from $G_1$ to $G_2$ has gained an energy $eMV \sin \omega t$. Here $M$ is the beam-coupling coefficient discussed in Chap. 3; $M \leqq 1$. Hence when the electron passes through $G_2$ it has a velocity $v$ given by the relation $mv^2/2 = eV_0 + eMV \sin \omega t$. Since $mv_0^2/2 = eV_0$, it follows that

$$v = v_0 \sqrt{1 + \left(\frac{MV}{V_0}\right) \sin \omega t} \approx v_0 \left[1 + \left(\frac{MV}{2V_0}\right) \sin \omega t + \cdots \right], \tag{2}$$

to a good degree of approximation when $MV/2V_0 \ll 1$.

It is this periodic variation of electron velocity that is expressed by saying that the beam is velocity-modulated[1] as it leaves the input gap between $G_1$ and $G_2$; the quantity $V/V_0$ is known as the "depth of modulation."

Region $C$—which extends from $G_2$ to the first grid $G_3$ of the output gap —is called the "drift space." It is assumed, again for simplicity, that in region $C$ there are no d-c fields and no r-f fields, and that any space-charge effects are negligible. The only effects are kinematic; the electrons that were speeded up in $B$ begin to catch up with the slower electrons that are ahead of them, and eventually result in a breaking up of the beam into groups or bunches. This process, known as "bunching,"[2] is illustrated in Fig. 9·2; here the relation between distance and time is

[1] It should be noted that "modulation," as used here, does not have the common connotation of superposition of further time variation on an already sinusoidally varying quantity; rather, the time variation is superimposed on a previously time-constant quantity, the electron velocity. These two senses of "modulation" make possible somewhat awkward expressions such as "the frequency modulation of velocity-modulation tubes." The nomenclature is, however, well established by usage.

[2] D. L. Webster, *Jour. App. Phys.*, **10**, 501 (1939).

shown for each of a series of typical electrons in what is known as an "Applegate diagram." The velocity modulation appears as a periodic change in the slope of the electron trajectories at the input gap; bunching corresponds to the convergence and eventual crossing of these trajectories. It may be noted that at $G_3$ the current is not uniform in time; instead, it has r-f components. It may be noted also that the larger $V/V_0$ is, the less drift length is required to produce a given degree of bunching, and that with an excessive amount of r-f voltage or of drift length the

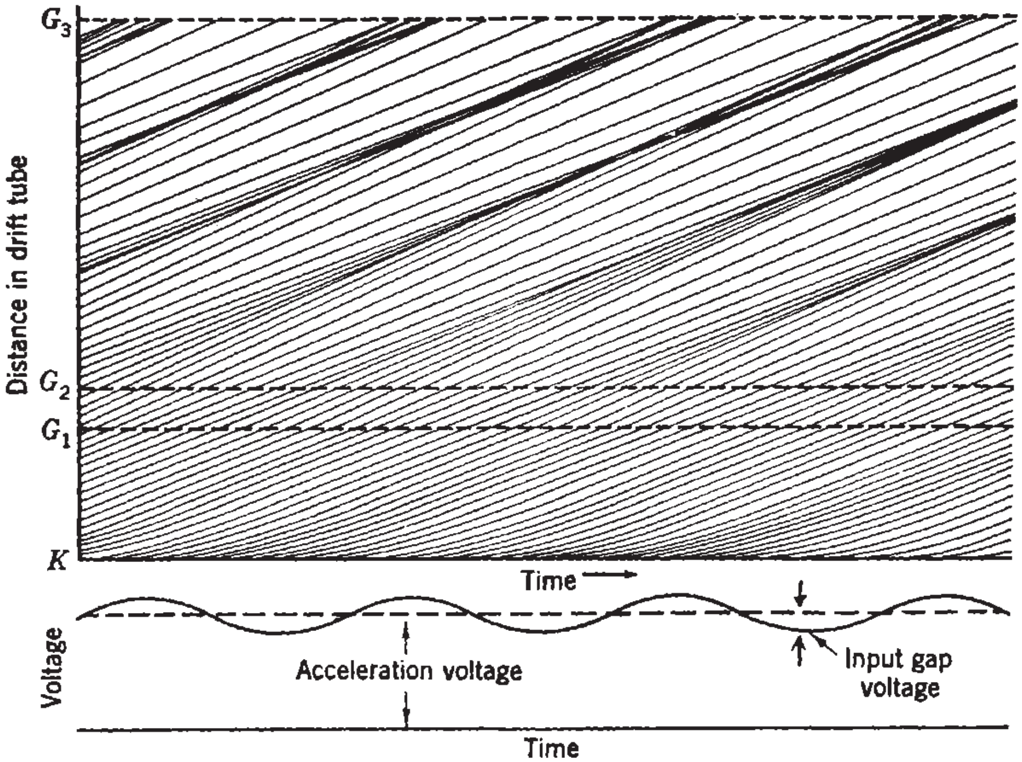


FIG. 9·2.—Applegate diagram of electron trajectories in velocity modulation and bunching.

trajectories diverge from their crossover points and the r-f component of current diminishes.

The main point is that the low-velocity cathode-grid control region of the triode is replaced in the klystron by a composite region in which external r-f control is exerted only on high-velocity electrons, and in which differences of finite electron transit times have been used to produce an intensity-modulated conduction current.

The simplifications assumed in the preceding description of velocity modulation and bunching are continued in the following quantitative discussion.

In considering the relation between time of departure from the input gap, $t_1$, and time of arrival at the output gap, $t_2$, the time of transit through the gaps is ignored. Then, by Eq. (2),

$$\omega t_2 = \omega t_1 + \frac{\omega l}{v} \approx \omega t_1 + \left(\frac{\omega l}{v_0}\right)\left[1 - \left(\frac{MV}{2V_0}\right) \sin \omega t_1\right].$$

The quantity $\omega l/v_0$ is the d-c transit time through the drift space, measured in radians of the input frequency; it is represented by $\theta_0$:

$$\theta_0 \equiv \frac{\omega l}{v_0}. \tag{3a}$$

It is also convenient to define

$$X_0 \equiv \frac{MV\theta_0}{2V_0}; \tag{3b}$$

$X_0$ as given here is a particular example of a dimensionless quantity known as the "bunching parameter"; the definition of the bunching parameter is generalized in succeeding sections. In terms of the transit angle $\theta_0$ and the bunching parameter $X_0$, the above transit-time relation becomes

$$\omega t_2 = \omega t_1 + \theta_0 - X_0 \sin \omega t_1. \tag{3c}$$

Many of the more general situations discussed later in this chapter are described by a transit-time relation given in the above form, but with a more general definition of bunching parameter than that given in Eq. (3*b*). In order to emphasize this fact, and in order to put the results of the discussion that now follows into a form that will be readily applicable later, the subscript is omitted from the bunching parameter in the discussion of the consequences of Eq. (3*c*).

This relation embodied in Eq. (3*c*) is shown in Fig. 9·3 for $X$ = 0.5, 1, 1.84, and 3.83. The quantitative relations in the bunching process are more clearly indicated here than in Fig. 9·2, and the illustration suggests a simple means of finding the actual waveform of the bunched current by application of the principle of conservation of charge. Thus, the electrons arriving at the output gap in the time interval $\Delta t_2$ are made up of one or more groups of electrons (three for the case indicated in Fig. 9·3) that have left the input gap during intervals $\Delta t_1 = |dt_1/dt_2|\,\Delta t_2$. If the d-c beam current is $I_0$, the total charge carried by the electrons arriving in $\Delta t_2$ is

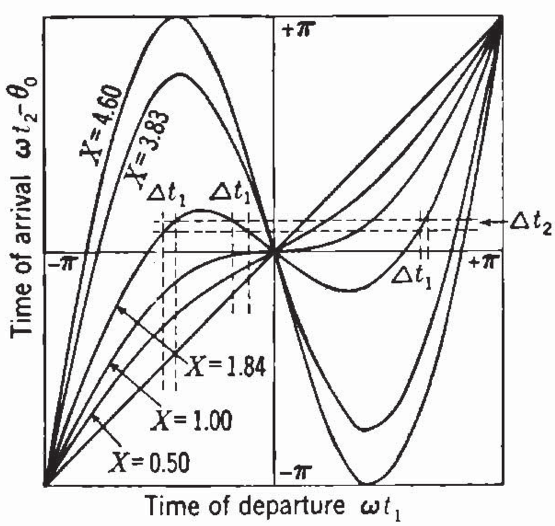


FIG. 9·3.—Relation between time of departure from input gap, $t_1$, and time of arrival at output gap, $t_2$, for several values of bunching parameter $X$.

$$I_0 \sum_{t_1(t_2)} \Delta t_1 = I_0\,\Delta t_2 \sum_{t_1(t_2)} \left|\frac{dt_1}{dt_2}\right|,$$

when the summation encompasses all times of departure $t_1$ that correspond to the same time of arrival $t_2$. The total charge is also $i\,\Delta t_2$ where $i$ is the instantaneous current through the output gap; hence

$$i(t_2) = I_0 \sum_{t_1(t_2)} \left|\frac{dt_1}{dt_2}\right|. \tag{4}$$

This equation simply states that the output gap current at any instant $t_2$ is obtained from Fig. 9·3 by adding the absolute values of all the inverse derivatives $dt_1/dt_2$ corresponding to the given time of arrival $t_2$. This process has been carried out in obtaining Fig. 9·4, which therefore shows the dependence on time of the instantaneous output-gap current for the four previously used values of the bunching parameter, $X = 0.5$, 1, 1.84, and 3.83.

The infinite-current peaks are a striking feature of Fig. 9·4 and arise in an obvious manner. For $X < 1$, the electrons that arrive at the output gap at any given instant are those that left the input gap at a single previous instant; for $X > 1$, on the other hand, it has already been noted that for a portion of a cycle the electrons that left the input gap at several different times arrive simultaneously at the output gap. As indicated in Fig. 9·3, this portion of the cycle begins and ends at the values of $t_2$ for which $dt_2/dt_1 = 0$; $dt_2/dt_1 = 0$ means that electrons leaving the input gap in an increment of time $dt_1$ arrive at the output gap in an infinitely shorter increment of time $dt_2$ and therefore constitute an instantaneously infinite current, carrying only a finite charge.

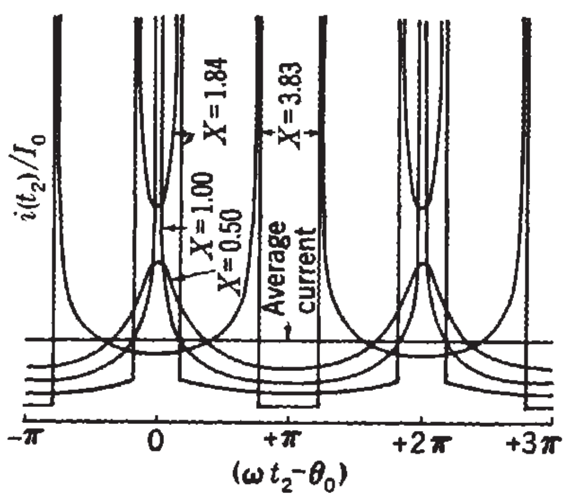


FIG. 9·4.—Dependence of relative current at output gap $i(t_2)/I_0$ on time $t_2$ for various values of bunching parameter $X$.

The component of the waveforms of Fig. 9·4 at the fundamental frequency depends not so much on the presence of infinite peaks as on the general concentration of current in one particular half cycle. It is obvious from Fig. 9·4 that this concentration increases as $X$ initially increases. As $X$ increases past unity the two infinite-current peaks, which contain a considerable concentration of current, become more and more separated in time. At $X = 1.84$ the concentration of current is still rather high; as may be seen shortly, this value of $X$ corresponds approximately to the maximum value of the fundamental component. At $X = 3.83$, however, the peaks are somewhat more than a half cycle apart and in their effect (for example, in driving a circuit) they oppose each other in phase; although the current is hardly constant in time, the fundamental component is exactly zero at this value of $X$.

For high harmonics the infinite-current peaks become very important because any one of such peaks provides an appreciable concentration of current in a half cycle of a high harmonic. Since infinite peaks occur only for $X \geqq 1$, not much harmonic content should be expected for $X < 1$. Whenever the two peaks that are present when $X > 1$ are separated by an integral number of half cycles of the harmonic in question, their resulting opposition in phase brings the content of this harmonic nearly to zero. The amplitude of higher harmonics is thus expected to be a maximum near $X = 1$, and to oscillate about zero as $X$ increases past this point.

The above description is an intuitive Fourier analysis of the bunched beam current; for more exact information an exact Fourier analysis is needed and this will now be made.

Since the output-gap current $i(t_2)$ is periodic with the angular frequency $\omega$, this current may be expressed as the sum of a series of harmonics of $\omega$:

$$i(t_2) = \operatorname{Re} \sum_{m=0}^{\infty} i_m e^{jm\omega t_2}. \tag{5}$$

The values of $i_m$ are thus the complex current amplitudes at the various harmonics, just as $V_1$ is the complex r-f gap-voltage amplitude. By the usual theory of Fourier series, the values of $i_m$ are given by

$$\pi i_m = \int_{-\pi}^{\pi} i(t_2) e^{-jm\omega t_2} d(\omega t_2). \tag{6}$$

If the relation for $i(t_2)$ given by Eq. (4) is recalled, it is apparent that

$$\pi i_m = I_0 \int_{-\pi}^{\pi} d(\omega t_2) e^{-jm\omega t_2} \sum_{t_1(t_2)} \left|\frac{dt_1}{dt_2}\right|. \tag{7}$$

This expression is made analytically inconvenient by the occurrence of the absolute value and discrete summation in the integrand. These features, arising from the multiple-valued dependence of $t_1$ on $t_2$ shown in Fig. 9·4, are necessary only for $X > 1$. For $X \leqq 1$, $|dt_1/dt_2|$ may be replaced by $dt_1/dt_2$, in which case the above equation becomes

$$\pi i_m = I_0 \int_{-\pi}^{\pi} e^{-jm\omega t_2}\, d(\omega t_1).$$

This equation has sometimes been derived for $X \leqq 1$ in this manner, and the results then applied to instances where $X > 1$. This procedure has given rise to some confusion, not because the equation is incorrect (it is not), but because its validity for $X > 1$ is not immediately obvious. The demonstration of this validity may be based explicitly on Eq. (7), but it may also be demonstrated in a more general manner as follows.

Since $i(t_2)\,dt_2$ is an element of charge, Eq. (6) is a summation of the phase factor $e^{-jm\omega t_2}$ over all electrons that pass through the output gap in one cycle. The order in which the contributions of the various electrons are summed up is immaterial; for example, a summation index not necessarily assigned in the order of arrival of electrons at the output gap may be associated with each individual electron. In this case,

$$i_m = \frac{\omega}{\pi} e \sum_n e^{-jm\omega t_{2n}},$$

where $e$ is the charge on the electron. Here $t_{2n}$ is the arrival time for the $n$th electron, and the summation is over all electrons passing through the output gap in one cycle. As a particular illustration, since $t_2$ is a single-valued function of $t_1$, $n$ may be identified with the time of departure $t_1$; since the electrons in the element of charge $I_0\,dt_1$ arrive (to first order) at the same time $t_2(t_1)$, the summation may be written as an integral, giving

$$i_m = \frac{1}{\pi} I_0 \int_{\omega t_2 = -\pi}^{\pi} e^{-jm\omega t_2(t_1)} d(\omega t_1).$$

Here the specific limits of integration indicate that the integral is extended only over those values of $t_1$ that, although they may not in themselves lie within a single period, correspond to arrival times $t_2$ lying within one period. But since $t_2 - t_1$ is a periodic function of $t_2$, the limits of integration may be further changed to correspond to an arbitrary addition or subtraction of an integral number of periods to the $t_1$ corresponding to any $dt_2$. In particular, this arbitrary change can be carried out in such a way as to make the integration over $t_1$ correspond to integration over a single consecutive period of $t_1$. This process is easily visualized with the aid of an extension of Fig. 9·3 to cover several periods of $t_1$ and $t_2$. Thus finally

$$\pi i_m = I_0 \int_{-\pi}^{\pi} e^{-jm\omega t_2} d(\omega t_1). \tag{8}$$

By Eq. (3), Eq. (8) may be written

$$i_m = \frac{I_0}{\pi} e^{-jm\theta_0} \int_{-\pi}^{\pi} e^{-jm(\omega t_1 - X \sin \omega t_1)} d(\omega t_1).$$

Using the Bessel function expansion of the integrand,

$$\int_{-\pi}^{\pi} e^{-jm(Z - X \sin Z)}\, dZ = 2\pi J_m(mX),$$

the equation for $i_m$ becomes

$$i_m = 2I_0 e^{-jm\theta_0} J_m(mX). \tag{9}$$

In Fig. 9·5 are shown, for the fundamental and several harmonics, the absolute values of the current components divided by beam current, $|i_m|/I_0 = 2J_m(mX)$. These curves show in more detail the dependence of current component on bunching parameter that has already been qualitatively discussed.

Since the leading term in $J_m(mX)$, and hence the predominant term for $X \ll 1$, is proportional to $X^m$, only the fundamental component is linear in $X$ for small bunching voltages. The maximum value of $|i_1|/I_0$, 1.16, occurs for $X = 1.84$ and, as the harmonic order increases, the value of $X$ for maximum harmonic content approaches unity. For $m \gg 1$ the maximum value of $J_m(mX)$ approaches the value $0.65/m^{1/3}$; this remarkably slow diminution of harmonic amplitude with harmonic order is a characteristic feature of klystron bunching arising from the infinite peak of Fig. 9·4, as has already been noted qualitatively.

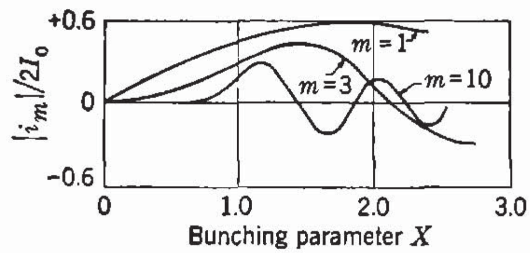


FIG. 9·5.—Dependence of harmonic components $i_m$ of bunched beam current on bunching parameter X for several values of harmonic order.

**9·3. Debunching in a Klystron.**—The preceding section has dealt with bunching as a process involving simply the kinematics of electrons in a field-free drift space. It is clear, however, that with sufficiently high current density, space-charge forces may influence the electron motion more than the electrode or gap voltages. If this is true, it might be better to begin by considering bunching as a phenomenon involving waves in a traveling space charge.[1] The present discussion is concerned only with those effects of space charge that are easily considered as modifications of bunching, or as "debunching."[2] This distinction is not a sharp one and lies primarily in the degree of approximation.

*Space-charge Spreading of an Unneutralized D-c Beam.*—As an introduction to debunching, the orders of magnitude involved in space-charge

[1] D. L. Webster, *Jour. App. Phys.*, **10**, 15 (1939).
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S. Ramo, *Proc. I.R.E.*, **27**, 757 (1939).
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E. Feenberg and D. Feldman, *Jour. App. Phys.*, **17**, 1025 (1946).

[2] E. Feenberg, "Theory of Bunching," Sperry Gyroscope Co. Report 5221–105, Nov. 24, 1942.
W. W. Hansen, unpublished notes.