Lecture 5

High Power Microwave Sources
EEC746

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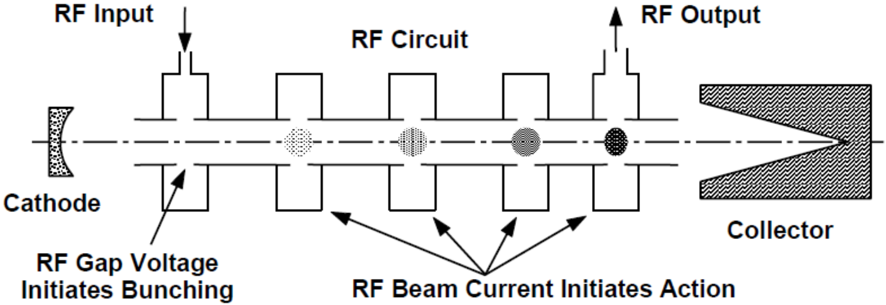
Beam Gap Interaction

1. Induced Current
   - Total Current, Continuity of Total Current
   - Schockley-Ramo Theorem

Beam Gap Interaction



Outline

1. Induced Current
   - Total Current, Continuity of Total Current
   - Schockley-Ramo Theorem

Definition of Total Current

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}_c + \mathbf{J}_d$$

- $\mathbf{J}_c$ is the convection current (due to motion of charges).
- $\mathbf{J}_d$ is the displacement current (due to time variation of the electric field).

Definition

Total current density $\mathbf{J}_t$ is defined as,

$$\mathbf{J}_t \equiv \mathbf{J}_c + \mathbf{J}_d$$

Continuity of total current

$$\nabla \cdot \mathbf{J}_t = 0$$

Outline

1. Induced Current
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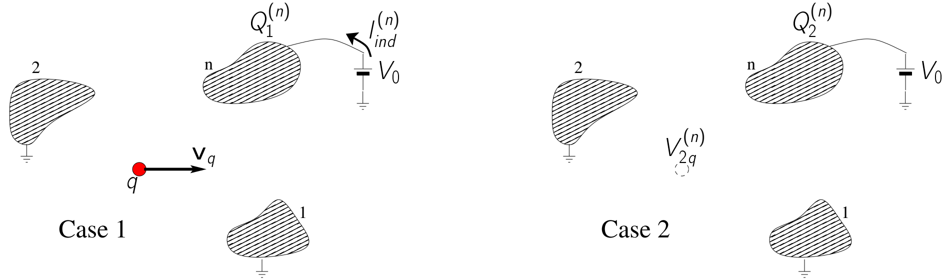
Schockley-Ramo Theorem



Green's Theorem for the two cases potential functions V_1 and V_2,

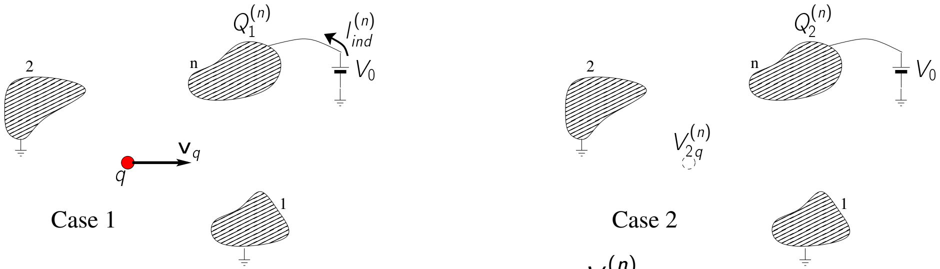
$$\int_V \left(V_1 \nabla^2 V_2 - V_2 \nabla^2 V_1\right) d\tau = \oint_S \left(V_1 \nabla V_2 - V_2 \nabla V_1\right) \cdot \mathbf{n} da$$

$$\nabla^2 V_1 = -\frac{q}{\varepsilon_0} \delta^3 \left(\mathbf{r} - \mathbf{r}_q\right), \qquad \nabla^2 V_2 = 0$$

$$q V_{2q}^{(n)} = Q_1^{(n)} V_0 - Q_2^{(n)} V_0$$

$$Q_{ind}^{(n)} = Q_2^{(n)} - Q_1^{(n)} = -q \frac{V_{2q}^{(n)}}{V_0}$$

Schockley-Ramo Theorem



$$Q_{ind}^{(n)} = Q_2^{(n)} - Q_1^{(n)} = -q\frac{V_{2q}^{(n)}}{V_0}$$

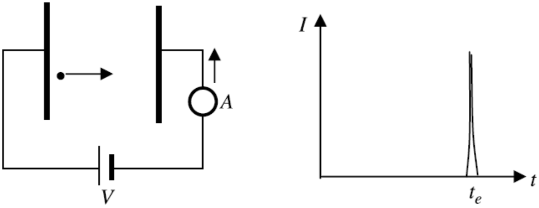
$$I_{ind}^{(n)} = \frac{dQ_{ind}^{(n)}}{dt} = -\frac{q}{V_0}\mathbf{v}_q \cdot \nabla V_{2q}^{(n)} = \frac{1}{V_0} q\mathbf{v}_q \cdot \mathbf{E}_q$$

For a group of charges,

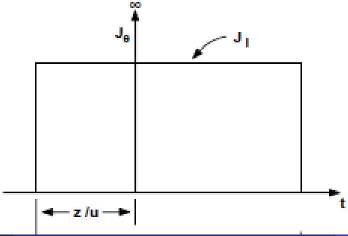
$$I_{ind}^{(n)} = \frac{1}{V_0}\sum_q q\mathbf{v}_q \cdot \mathbf{E}_q = \frac{1}{V_0}\int_V \mathbf{J}_c \cdot \mathbf{E}\, d\tau$$

$$\boxed{I_{ind}^{(n)} = \frac{1}{V_0}\int_V \mathbf{J}_c \cdot \mathbf{E}\, d\tau}$$

Electron Bunch with Uniform Velocity



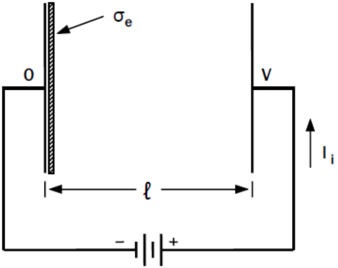
$$I_i = \frac{1}{V}\int_V \mathbf{J}_c \cdot \mathbf{E}\, d\tau, \qquad I_i = \frac{1}{-E\ell}(-\sigma)\, uEA = \frac{\sigma}{\ell} uA$$

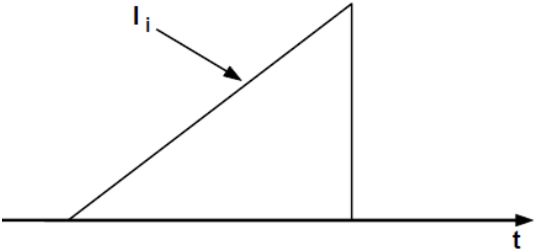


Change in Velocity is Considered

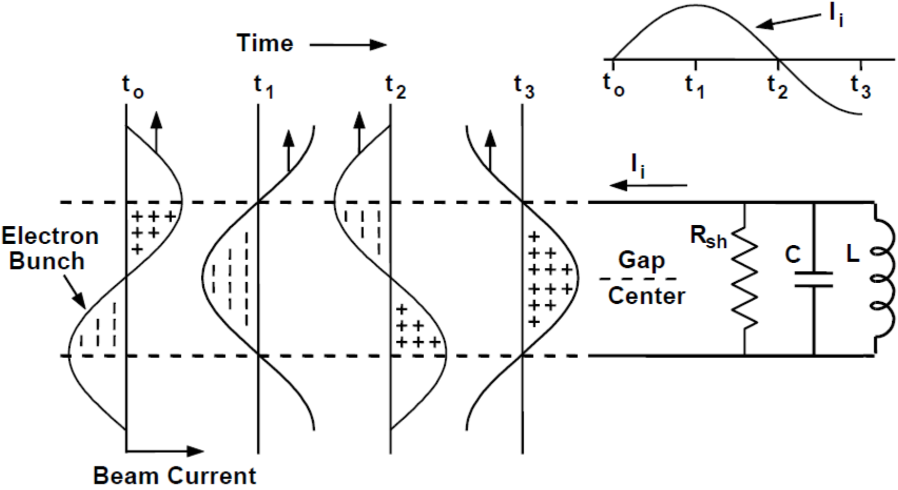
$$u = \eta \frac{V}{\ell} t,$$

$$I_i = \frac{1}{V} \int \mathbf{J}_c \cdot \mathbf{E} d\tau = \frac{1}{V} \eta (-\sigma) \eta \frac{V}{\ell} t \times \left(-\frac{V}{\ell} \right) A = \sigma \eta \frac{V}{\ell^2} At$$

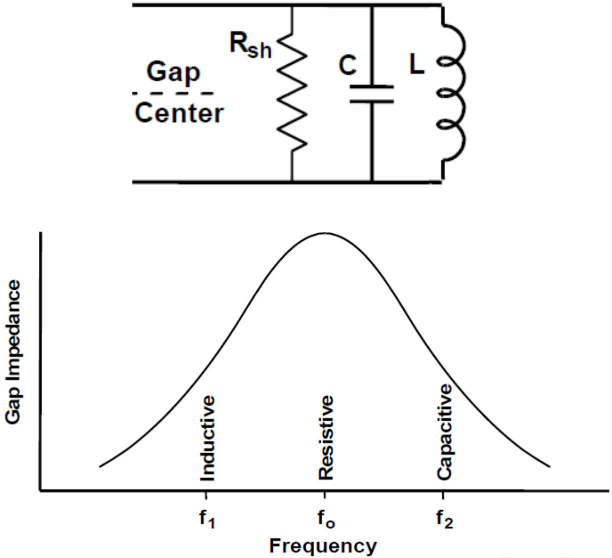




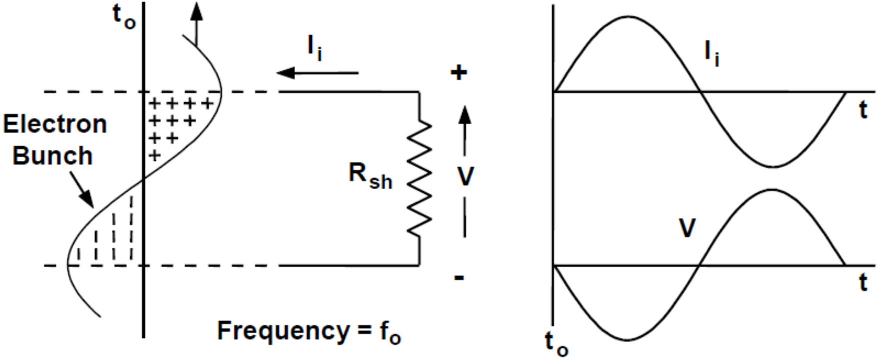
Current Induced in Cavity Circuit



Impedance Presented to the Beam as a Function of Frequency



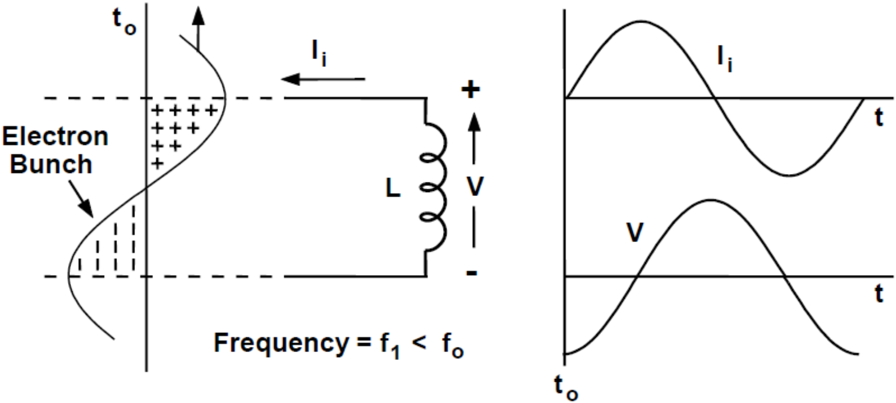
Voltage Induced with Resistive Loading



$$I_i = I_{\max} \sin \omega t, \qquad V = -I_i R = -I_{\max} R \sin \omega t$$

- The induced voltage slightly decelerates the electron bunches.
- A new signal is generated in the beam which is 90° with the current I_i.

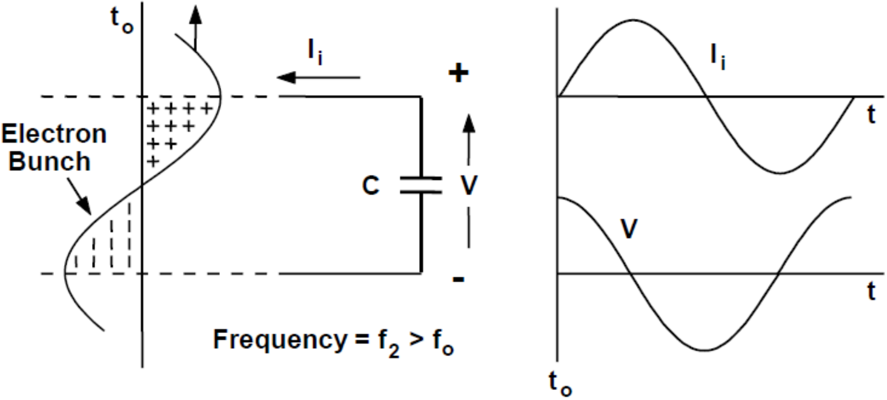
Voltage Induced with Inductive Loading



$$I_i = I_{\max} \sin \omega t, \qquad V = \Im \left\{ -j\omega L I_{\max} e^{j\omega t} \right\} = -\omega L I_{\max} \cos \omega t$$

The induced voltage decelerates the electrons in and near the leading edge and accelerates those in and near the trailing edge, hence enhances the beam bunching.

Voltage Induced with Capacitive Loading



$$I_i = I_{\max} \sin \omega t, \qquad V = \Im \left\{ \frac{-1}{j\omega C} I_{\max} e^{j\omega t} \right\} = \frac{I_{\max}}{\omega C} \cos \omega t$$

The induced voltage accelerates electrons away from the leading and trailing edges of a bunch, which tends to destroy the bunch (debunching).