Lecture 4

High Power Microwave Sources
EEC746

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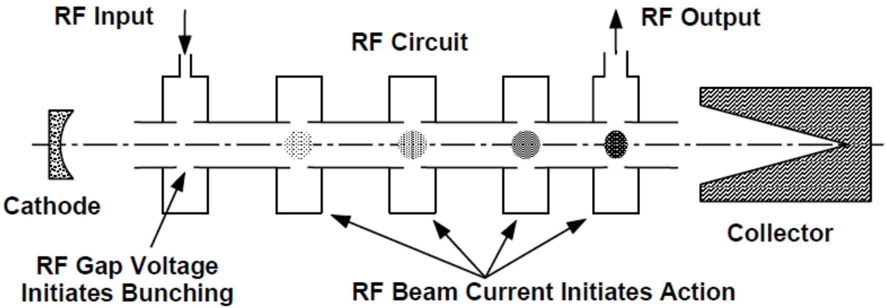
1. Beam Modulation
   - Gridded (Planar) Gaps
   - Gridless (Nonplanar) Gap

Beam Gap Interaction

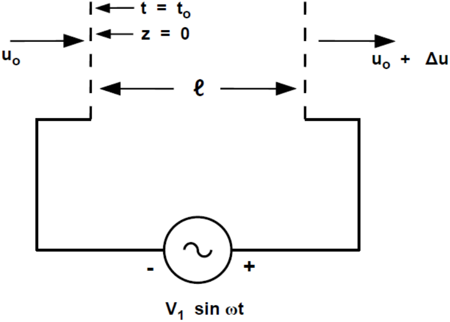


Outline

1. Beam Modulation
   - Gridded (Planar) Gaps
   - Gridless (Nonplanar) Gap

Gridded (Planar) Gaps

The equation of motion for electrons in the gap is,



$$\frac{d^2z}{dt^2} = \eta \frac{V_1}{\ell} \sin \omega t,$$

$$u - u_0 = \eta \frac{V_1}{\omega \ell} \left[\cos \omega t_0 - \cos \omega \left(t_0 + T\right)\right]$$

$$u - u_0 = 2\eta \frac{V_1}{\omega \ell} \sin\left(\omega T/2\right) \sin \omega \left(t_0 + \frac{T}{2}\right), \qquad \text{where } T = \frac{\ell}{u_0}$$

$$u = u_0 \left[1 + \frac{\alpha M_p}{2} \sin \omega \left(t_0 + \frac{T}{2}\right)\right] = u_0 \left[1 + \frac{\alpha M_p}{2} \sin \omega \left(t_1 - \frac{T}{2}\right)\right]$$

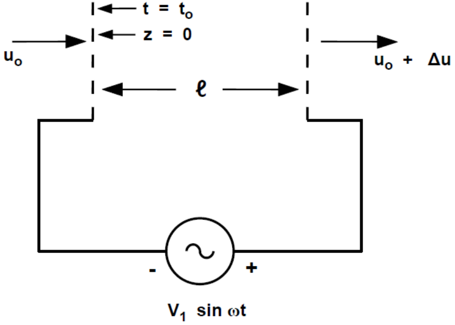
Gridded (Planar) Gaps

The equation of motion for electrons in the gap is,

$$u = u_0 \left[1 + \frac{\alpha M_p}{2} \sin \omega \left(t_0 + \frac{T}{2} \right) \right],$$
$$= u_0 \left[1 + \frac{\alpha M_p}{2} \sin \omega \left(t_1 - \frac{T}{2} \right) \right]$$

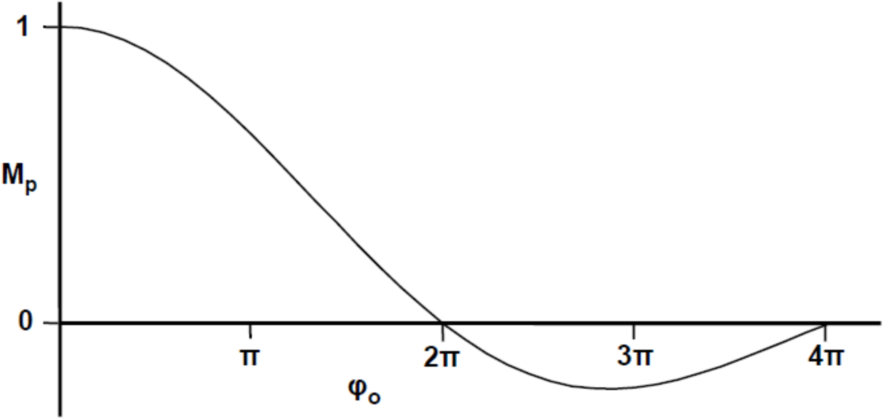
where,

- the *depth of modulation* $\alpha = V1/V0$
- the *gap coupling coefficient* $M_p = \dfrac{\sin(\omega T/2)}{\omega T/2}$.

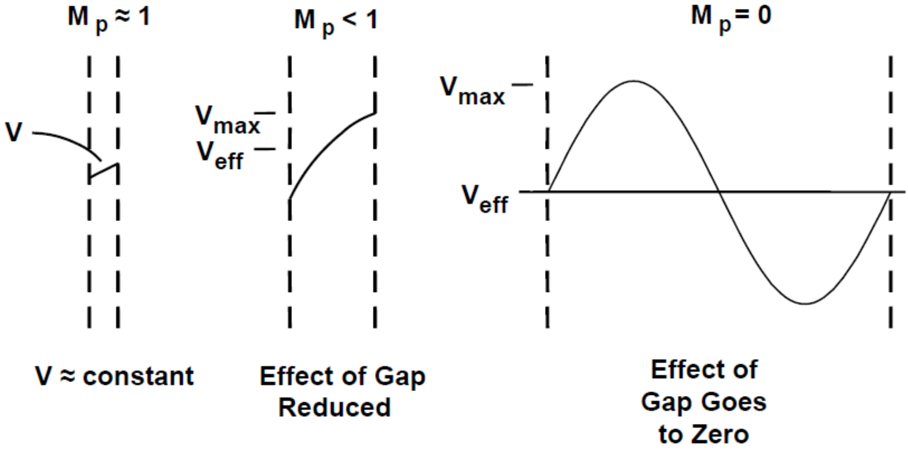


- the *transit time and phase* are $T = \dfrac{\ell}{u_0}$ and $\varphi_0 = \omega T = \dfrac{\omega \ell}{u_0}$, respectively.
- entrance and exit times to the cavity are t_0 and t_1, respectively $(t_1 = t_0 + T)$.

Gap Coupling Coecient M_p



$$M_p = \frac{\sin\left(\varphi_0/2\right)}{\varphi_0/2}, \qquad \varphi_0 = \omega T = \frac{\omega \ell}{u_0}$$


M p ≈ 1
V
V ≈ constant
M p < 1
Vmax
Veff
Effect of Gap Reduced
M p = 0
Vmax
Veff
Effect of Gap Goes to Zero

Outline

1. Beam Modulation
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Assuming small signal variation for the electron velocity such that we can write,

$u = u_0 + \Im\left\{u_p(z)e^{j\omega t}\right\}$, where $u_p(z)$ is the phasor of AC part of velocity.

$$\frac{du}{dt} = \Im\left\{\left[j\omega u_p(z) + u_0\frac{\partial u_p(z)}{\partial z}\right]e^{j\omega t}\right\}$$

Similarly the z-component of the electric field,

$$E = \Im\left\{E_p(z)\,e^{j\omega t}\right\}.$$

The equation of motion

$$\frac{du}{dt} = \Im\left\{\left[j\omega u_p(z) + u_0\frac{\partial u_p(z)}{\partial z}\right]e^{j\omega t}\right\} = -\eta\Im\left\{E_p(z)\,e^{j\omega t}\right\}$$

$$\frac{\partial u_p(z)}{\partial z} + j\beta_e u_p(z) = -\frac{\eta}{u_0}E_p(z)$$

$$u_p(z) = -\frac{\eta}{u_0}e^{-j\beta_e z}\int_{z_0}^{z}E_p(\zeta)\,e^{j\beta_e\zeta}\,d\zeta$$

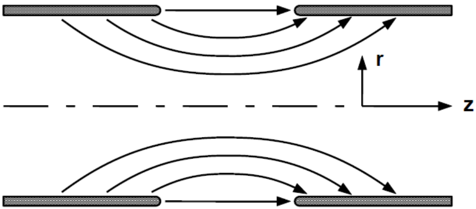
Gridless (Nonplanar) Gap

$$u_p(z) = -\frac{\eta}{u_0} e^{-j\beta_e z} \int_{z_0}^{z} E_p(\zeta)\, e^{j\beta_e \zeta}\, d\zeta$$

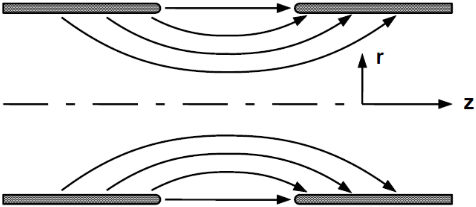
As z_0 goes to $-\infty$ and z goes to ∞ (very large away of the gridless gap discontinuity),

$$\boxed{u_p(z) = -\frac{\eta}{u_0} e^{-j\beta_e z} \int_{-\infty}^{\infty} E_p(r,\zeta)\, e^{j\beta_e \zeta}\, d\zeta}$$

$$\boxed{u_p(z) = -\frac{\eta}{u_0} e^{-j\beta_e z} \tilde{E}_p(r,\beta_e),} \qquad \text{where} \qquad \tilde{E}_p(r,k) = \int_{-\infty}^{\infty} E_p(r,\zeta)\, e^{jk\zeta}\, d\zeta$$



Electric Field in the Gridless Gap



$$\tilde{E}_p(r,k) = \int_{-\infty}^{\infty} E_p(r,z)\, e^{jkz}\, dz, \qquad E_p(r,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}_p(r,k)\, e^{-jkz}\, dk$$

The field $E_p(r,z)$ satisfy the equation,

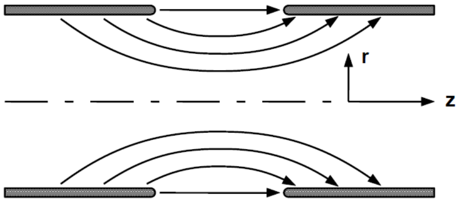
$$\frac{\partial^2 E_p}{\partial r^2} + \frac{1}{r}\frac{\partial E_p}{\partial r} + \frac{\partial^2 E_p}{\partial z^2} + k_0^2 E_p = 0$$

$$\frac{\partial^2 \tilde{E}_p}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{E}_p}{\partial r} - K^2 \tilde{E}_p = 0, \qquad \text{where } K^2 = k^2 - k_0^2$$

$$\boxed{\tilde{E}_p(r,k) = \tilde{E}_p(0,k)\, I_0(Kr)}$$

Coupling Coefficient

The coupling coefficient at radius r,



$$M(r) = \frac{\int_{-\infty}^{\infty} E_p(r,z)\, e^{j\beta_e z}\, dz}{\int_{-\infty}^{\infty} E_p(a,z)\, dz}$$

$$= \frac{\tilde{E}_p(r,\beta_e)}{\int_{-\infty}^{\infty} E_p(a,z)\, dz}$$

$$\because\ \tilde{E}_p(r,k) = \tilde{E}_p(0,k)\, I_0(Kr) \qquad \therefore\ \tilde{E}_p(r,\beta_e) = \tilde{E}_p(a,\beta_e)\, \frac{I_0(K_e r)}{I_0(K_e a)},$$

$$M(r) = \frac{\tilde{E}_p(r,\beta_e)}{\int_{-\infty}^{\infty} E_p(a,z)\, dz} = \frac{\tilde{E}_p(a,\beta_e)}{\int_{-\infty}^{\infty} E_p(a,z)\, dz}\, \frac{I_0(K_e r)}{I_0(K_e a)}$$

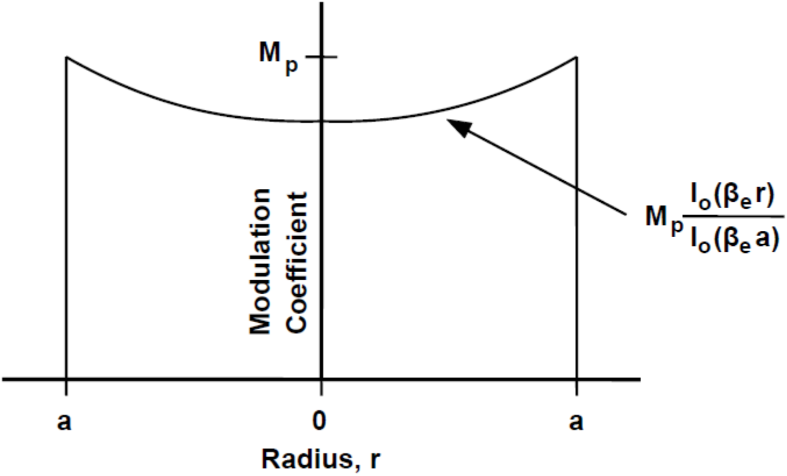
$$M(r) = M(a)\frac{I_0(K_e r)}{I_0(K_e a)}, \qquad \text{where } K_e^2 = \beta_e^2 - k_0^2 \approx \beta_e^2$$

At $r = 0$,

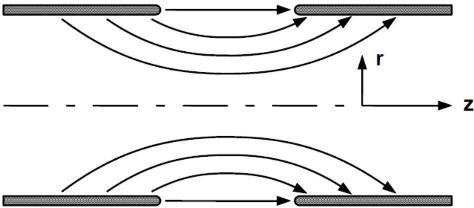
$$M(0) = \frac{M(a)}{I_0(\beta_e a)}$$

Coupling Coefficient

$$M(r) = M(a)\frac{I_0(K_e r)}{I_0(K_e a)}$$



Gridless (Nonplanar) Gap



$$\boxed{u_p(z) = -\frac{\eta}{u_0} e^{-j\beta_e z} \tilde{E}_p(0, \beta_e),}$$

where $\tilde{E}_p(0,k) = \int_{-\infty}^{\infty} E_p(0,\zeta)\, e^{jk\zeta} d\zeta$

The gap voltage V_1 is given by,

$$V_1 = -\int_{-\infty}^{\infty} E_p(a,z)\, dz$$

$$u_p(z) = -\frac{\eta}{u_0} e^{-j\beta_e z} \tilde{E}_p(0,\beta_e) = \frac{\eta V_1}{u_0} e^{-j\beta_e z} M(0)$$

$$u_p(z) = u_0 \frac{V_1}{2V_0} e^{-j\beta_e z} M(0) = u_0 \frac{\alpha M(0)}{2} e^{-j\beta_e z}$$

$$u = u_0 + \Im\left\{u_p(z) e^{j\omega t}\right\} = u_0 \left[1 + \frac{\alpha M(a)}{2 I_0(\beta_e a)} \sin(\omega t - \beta_e z)\right]$$