Multichannel modulation

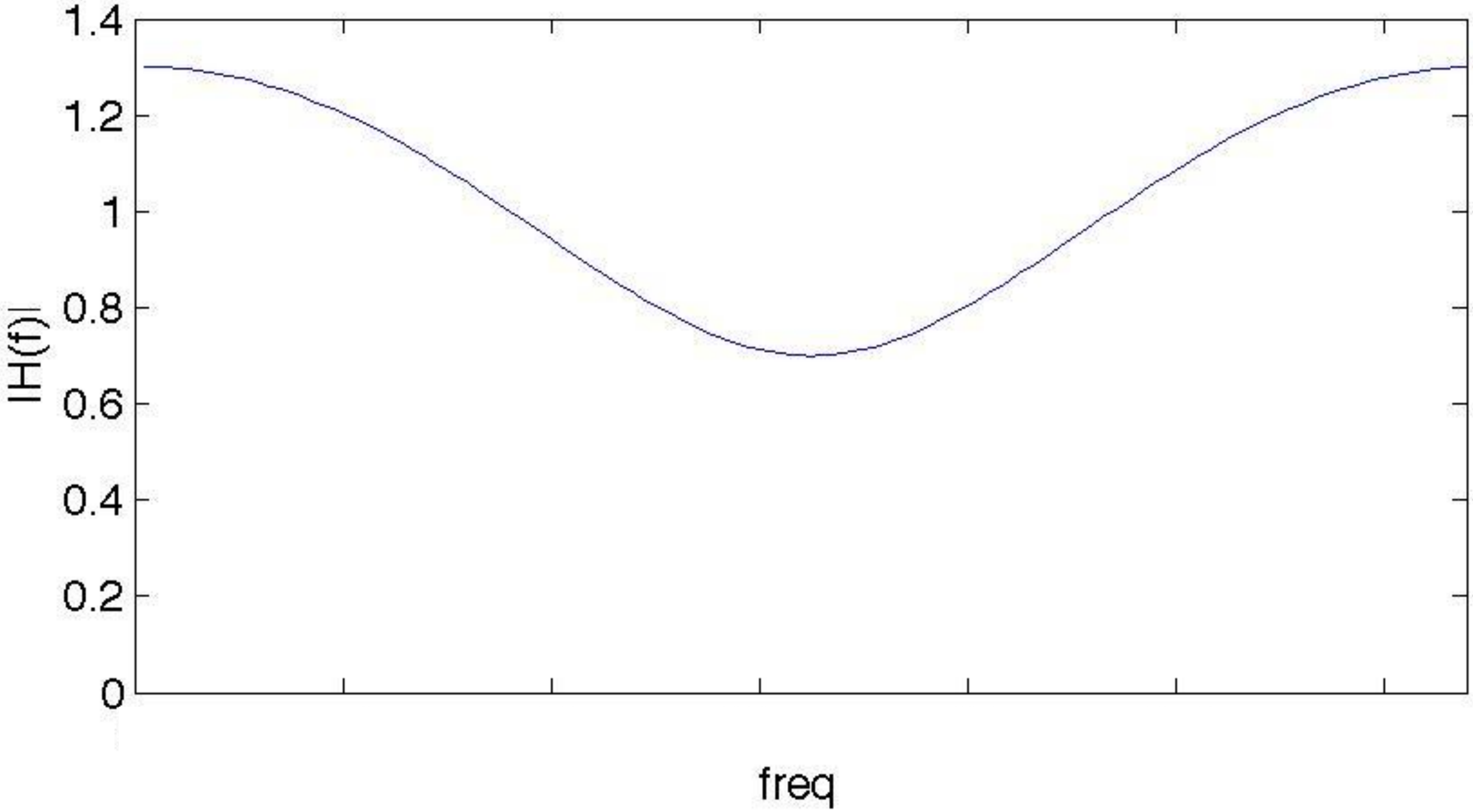
Motivation

- For a twisted pair, the channel introduces ISI which is proportional to the data rate.
- Multichannel modulation uses the principle of divide & conquer.
- Divide the wideband channel with severe ISI into smaller channels that can be considered AWGN channels.

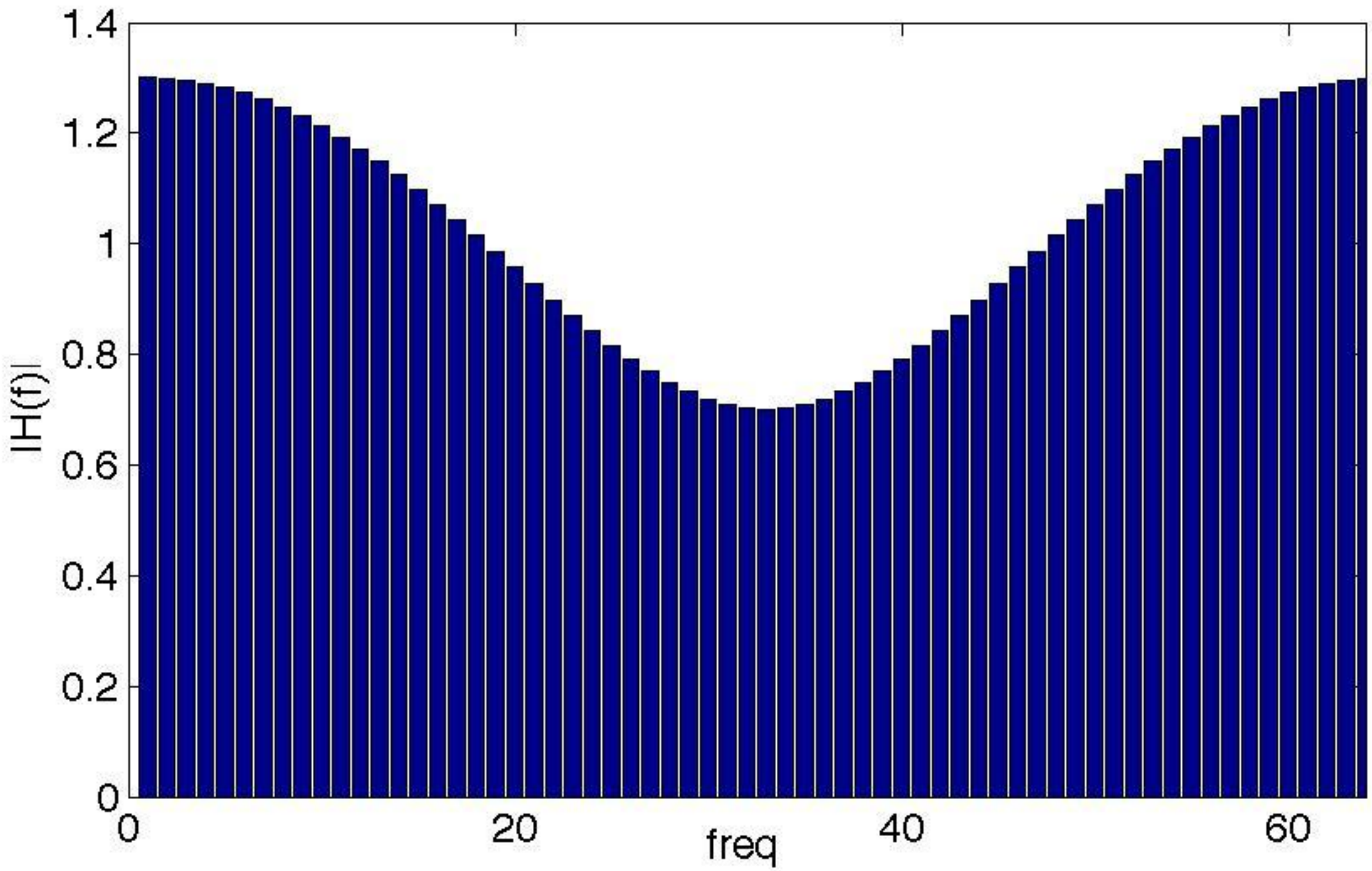
Approach

- Data T_x over difficult channel is transformed through the use of advanced DSP techniques into parallel T_x of the given data stream over a large number of subchannels which can be considered AWGN.

Fading Channel

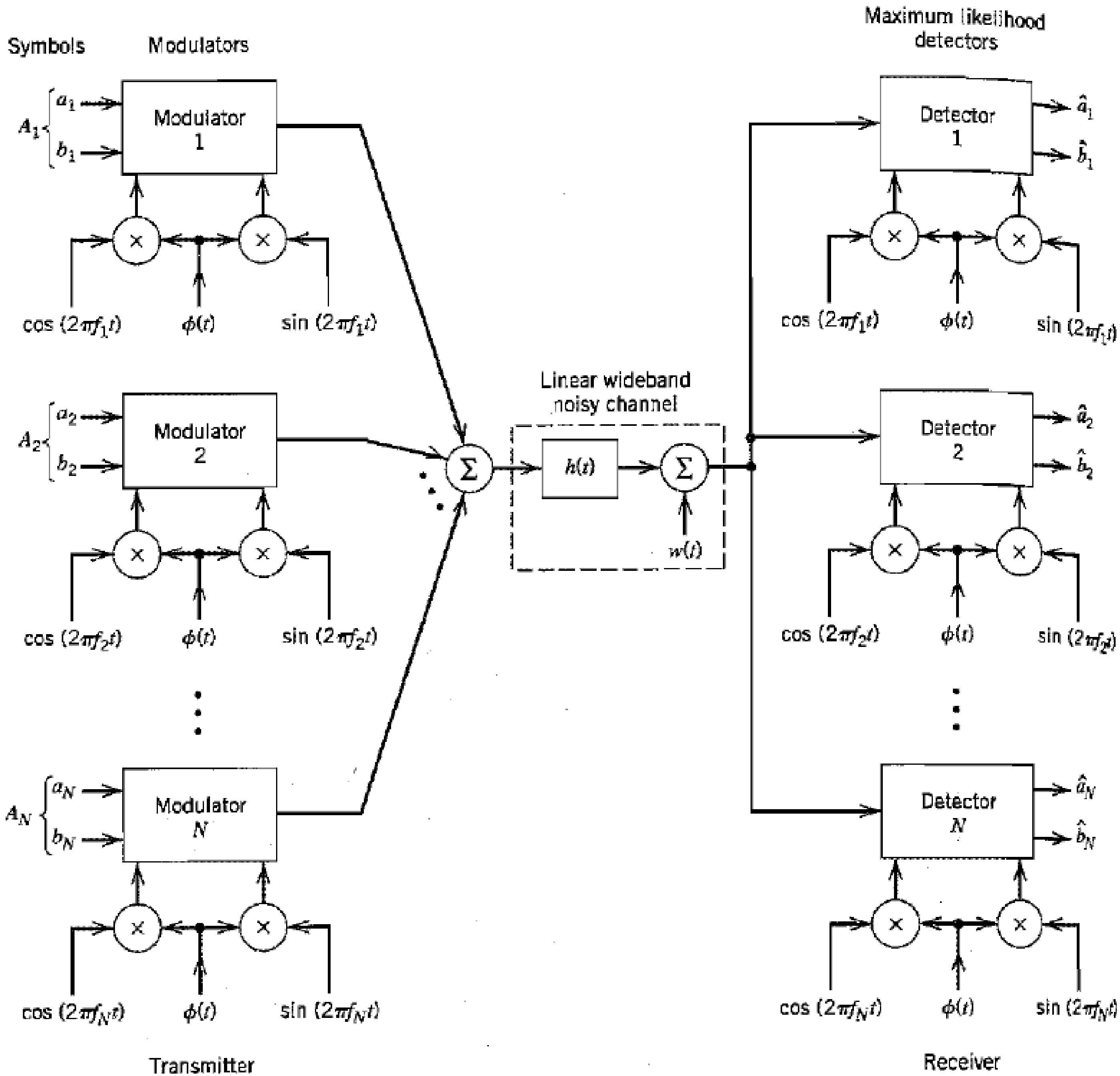


Dividing the channel



MC system

- Data T_x over each subchannel can be independently & individually optimized.
- The need for complicated equalization of a wideband channel is replaced by the need of multiplexing & demultiplexing the T_x of the incoming data stream over a large number of narrow band channels.
- The complexity of multicarrier system is high but FFT made it possible.

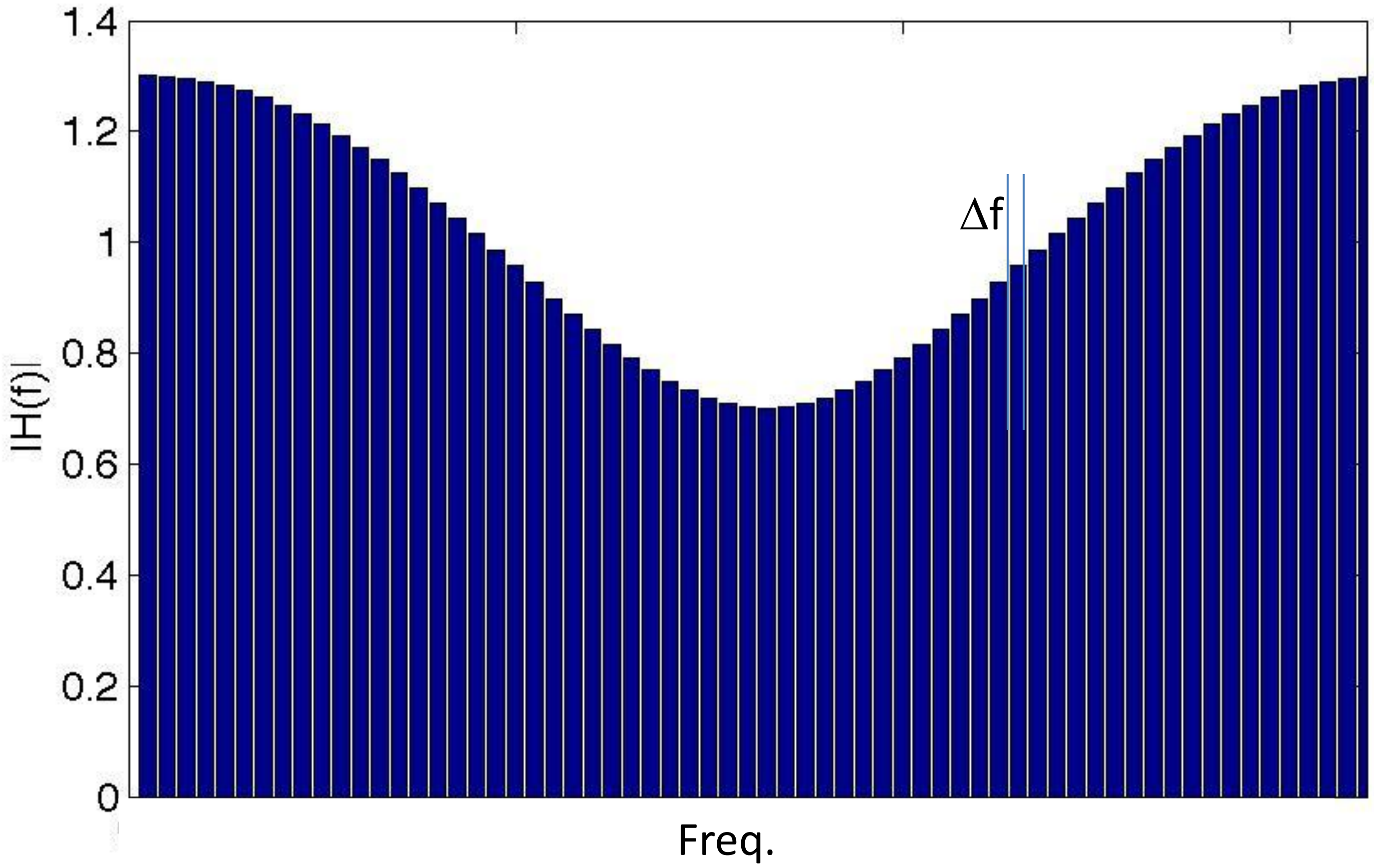


Symbols
Modulators
A_1
a_1
b_1
Modulator 1
$\cos(2\pi f_1 t)$
$\phi(t)$
$\sin(2\pi f_1 t)$
A_2
a_2
b_2
Modulator 2
$\cos(2\pi f_2 t)$
$\phi(t)$
$\sin(2\pi f_2 t)$
A_N
a_N
b_N
Modulator N
$\cos(2\pi f_N t)$
$\phi(t)$
$\sin(2\pi f_N t)$
Transmitter
Linear wideband noisy channel
Σ
$h(t)$
Σ
$w(t)$
Maximum likelihood detectors
Detector 1
$\hat{a}_1$
$\hat{b}_1$
$\cos(2\pi f_1 t)$
$\phi(t)$
$\sin(2\pi f_1 t)$
Detector 2
$\hat{a}_2$
$\hat{b}_2$
$\cos(2\pi f_2 t)$
$\phi(t)$
$\sin(2\pi f_2 t)$
Detector N
$\hat{a}_N$
$\hat{b}_N$
$\cos(2\pi f_N t)$
$\phi(t)$
$\sin(2\pi f_N t)$
Receiver

MC system

- The incoming binary data stream is first applied to a demux, thereby producing a set of N sub-streams.
- Each sub-stream represents a sequence of 2-elemnt sub-symbol which, for the symbol interval $0<t<T$, is denoted by (a_n,b_n), $n=1,2,3\ldots,N$
- The carrier frequency f_n of the n^{th} modulator is an integer multiple of the symbol rate $1/T$, $f_n=n/T$

Loading of MC systems



MC system

$$\phi(t) = \sqrt{\frac{2}{T}}\mathrm{sinc}\left(\frac{\mathrm{t}}{\mathrm{T}}\right), \qquad -\infty < t < \infty$$

- For each n, the two quadrature-modulated sinc functions form an orthogonal pair

$$\int_{-\infty}^{\infty} (\phi(t)\cos(2\pi f_n t))(\phi(t)\sin(2\pi f_n t))dt = 0 \quad \text{for all } n$$

Properties of basis functions

- Recognize that $\exp(2\pi f_n t) = \cos(2\pi f_n t) + j\sin(2\pi f_n t)$
- We may completely redefine the passband basis function in the complex form

$$\left\{ \frac{1}{\sqrt{2}} \phi(t) \exp(2\pi f_n t) \right\}, \qquad n = 1, 2, \ldots, N$$

- Hence, the passband basis functions form an orthnormal set

$$\int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{2}} \phi(t) \exp(2\pi f_n t) \right) \left(\frac{1}{\sqrt{2}} \phi(t) \exp(2\pi f_k t) \right)^* dt = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases}$$

Properties

- The set of channel-output functions $h(t)*\phi(t)$ remains orthogonal for a linear channel with arbitrary impulse response $h(t)$, * denotes convolution
- The channel is thus partitioned into a set of independent subchannels operating in continuous time

Capacity of AWGN channels

- $C=B \log_2(1+SNR)$ bps
- $C=1/2 \log_2(1+SNR)$ bits/symbols
- C can only be reached with extremely complex coding technique for implementable systems.

Attainable rate R

- $R = 1/2 \log_2(1+SNR/\Gamma)$ bits/ symbol
- Γ=SNR gap
- For a SNR=255(24dB)→C=4 bits/symbol
- However, the rate that will achieve the required BER, using the implemented system, is only 2bits/symbol
- $2 = 1/2 \log_2(1+255/\Gamma)$
- $(1+255/\Gamma)=16$
- $\Gamma=17$

Loading of MC Trans. system

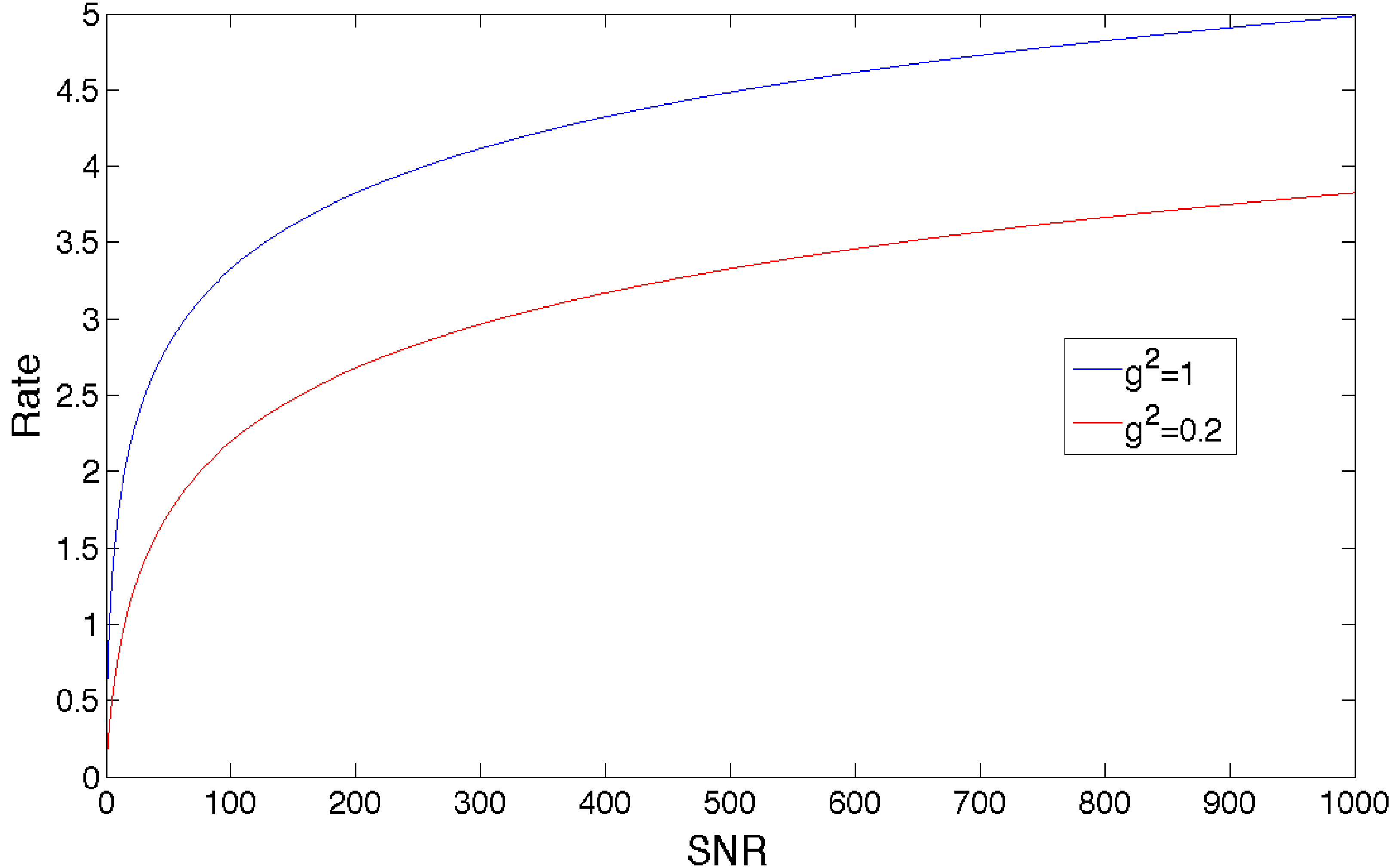
- Define $g_n = \left| H(f_n) \right|$
- Total rate that can be Tx on all channels

$$R = 1/2 \sum_{n=1}^{N} \log_2 \left(1 + \frac{g_n^2 P_n}{\sigma_n^2 \Gamma} \right)$$

- The noise variance σ_n^2 is $\Delta f N_0$ for all n.
- We want to maximize R through the proper allocation of the total Tx power among various channels for a constant total transmit power.

$$\sum_{n=1}^{N} P_n = P = \text{constant}$$

Capacity



Rate
SNR
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
100
200
300
400
500
600
700
800
900
1000
g²=1
g²=0.2

Optimization problem

- Maximize the bit rate for the entire MC transmission system through an optimal sharing of the total transmit power P between the N subchannels, subject to the constraint that P is maintained constant

Lagrange Multiplier λ

$$J = \frac{1}{2}\sum_{n=1}^{N}\log_2\left(1+\frac{g_n^2 P_n}{\Gamma\sigma_n^2}\right)+\lambda\left(P-\sum_{n=1}^{N}P_n\right)$$

using $\log_2(c) = \log_2(e)\log_e(c)$

and $\frac{\partial J}{\partial P_n} = 0$ we get $\frac{\frac{1}{2}\log_2(e)}{P_n + \frac{\Gamma\sigma_n^2}{g_n^2}} = \lambda$

Which can be reduced to

$$P_n + \frac{\Gamma\sigma_n^2}{g_n^2} = K, \quad \text{for } n = 1, 2, \ldots N$$

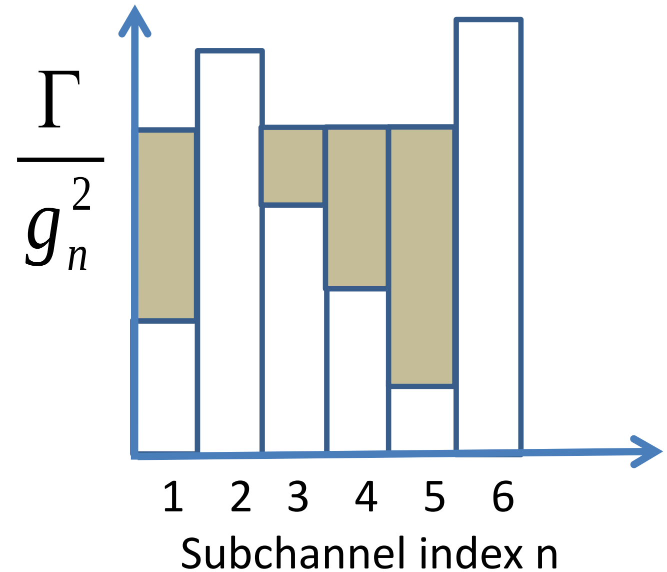
Optimum Allocation

- The sum of the transmit power and the noise variance scaled by Γ / g_n^2 must be maintained constant for each subchannel.
- The process of allocating P to the individual subcahnnel so as to maximize the bit rate of the entire multichannel transmission system is called *loading*

Water-Filling Interpretation

$$P_n + \frac{\Gamma \sigma_n^2}{g_n^2} = K, \quad \text{for } n = 1, 2, \ldots N \quad (*)$$

$$\sum_{n=1}^{N} P_n = P = \text{constant}$$



Assuming constant Γ and σ_n note the following:

-The sum of P_n and the scaled noise power satisfies (*) for 4 subchannel for a given P

-The sum of the 4 subchannels consumes P

-The remaining 2 subchannels have been eliminated because they would require negative power to satisfy (*) for K, which is unacceptable

Water-filling algorithm

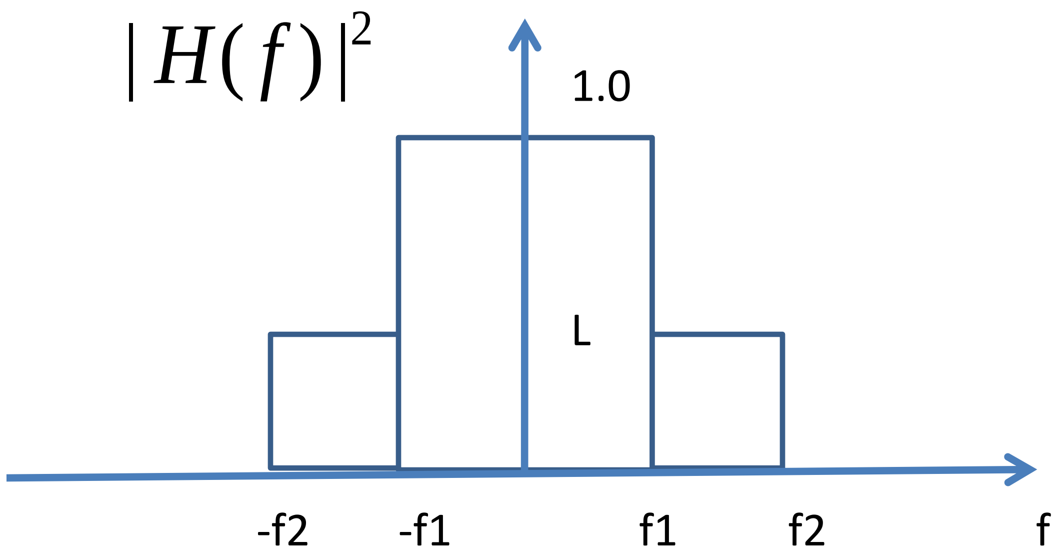
- The above solution is referred to as the water-filling solution.
- The terminology follows from the analogy with a fixed amount of water (P) being poured into a container with a number of connected regions each having different depth (noise).
- The water distributes itself in such a way that a constant water level is attained across the whole container.

Equation Form

$$\begin{bmatrix} 1 & 1 & \dots & 1 & 0 \\ 1 & 0 & & 0 & -1 \\ 0 & 1 & & 0 & -1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \\ K \end{bmatrix} = \begin{bmatrix} P \\ \frac{-\Gamma\sigma^2}{g_1^2} \\ \frac{-\Gamma\sigma^2}{g_2^2} \\ \vdots \\ \frac{-\Gamma\sigma^2}{g_N^2} \end{bmatrix}$$

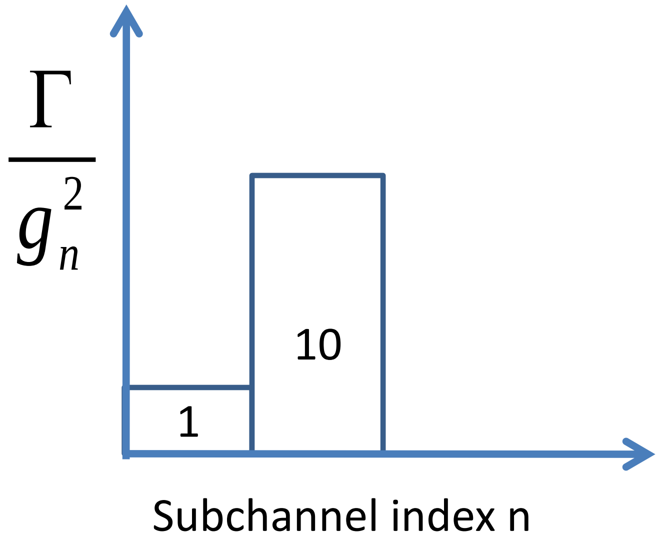
Example

Consider a linear channel whose squared magnitude response $|H(f)|^2$ has the piecewise linear form shown. Assume Γ=1 and σ^2=1. Calculate the optimum power allocation.



Example

- P1+P2=P
- P1-K=-1
- P2-K=-1/L
- P=10, L=0.1



Example

- K=10.5
- P1=9.5
- P2=0.5