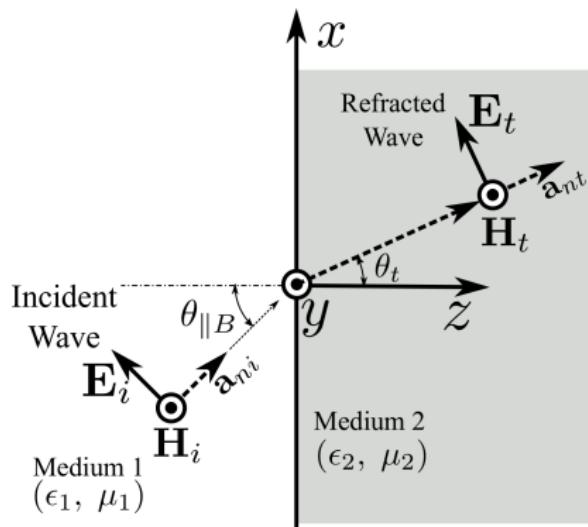


# Surface Palsmon Waves

Email: tamer@eng.cu.edu.eg

Electronics and Electrical Communications Department  
Faculty of Engineering  
Cairo University

# Surface Wave on Plasma Dielectric Boundary



- Brewster angle is obtained when,

$$\Gamma_{\parallel} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_{B\parallel}}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_{B\parallel}} = 0$$

$$\eta_1 \cos \theta_{B\parallel} = \eta_2 \cos \theta_t$$

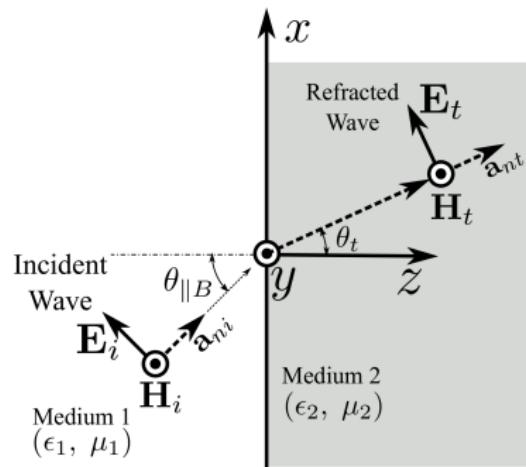
and using Snell's law of refraction:  $n_1 \sin \theta_{B\parallel} = n_2 \sin \theta_t$

$$\sin^2 \theta_{B\parallel} = \frac{1 - (\eta_2/\eta_1)^2}{1 - \left(\frac{n_1 \eta_2}{n_2 \eta_1}\right)^2} = \frac{1 - \frac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2}}{1 - \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}$$

For nonmagnetic material  $\mu_1 = \mu_2$ ,

$$\sin^2 \theta_{B\parallel} = \frac{1}{1 + \epsilon_1/\epsilon_2}$$

# Surface Wave on Plasma Dielectric Boundary



$$\sin^2 \theta_{B\parallel} = \frac{1}{1 + \epsilon_1/\epsilon_2},$$

$$\sin^2 \theta_t = \frac{\epsilon_1/\epsilon_2}{1 + \epsilon_1/\epsilon_2}$$

The angle  $\theta_{B\parallel}$  becomes non-real if  $\epsilon_1/\epsilon_2$  is not a positive value.

An interesting case would be if medium 2 is plasma for which,

$$\epsilon_2 = \epsilon_0 \left( 1 - \frac{\omega_p^2}{\omega^2} \right) < 0$$

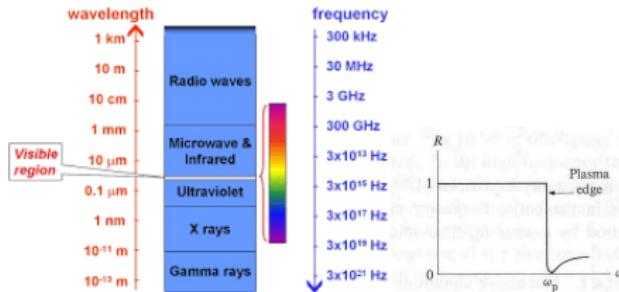
For the frequency range for which  $0 > \epsilon_1/\epsilon_2 > -1$ , the angle  $\theta_{B\parallel}$  is no longer real. However  $\sin \theta_{B\parallel}$  is pure real and  $\cos \theta_{B\parallel}$  is pure imaginary.

# Metal Plasma Frequency

Metal can be treated as a collection of free electrons with high density where the equivalent relative dielectric constant is given by Drude model

$$\epsilon_r = 1 + \frac{\omega_p^2}{-\omega^2 + j\omega\omega_d}$$

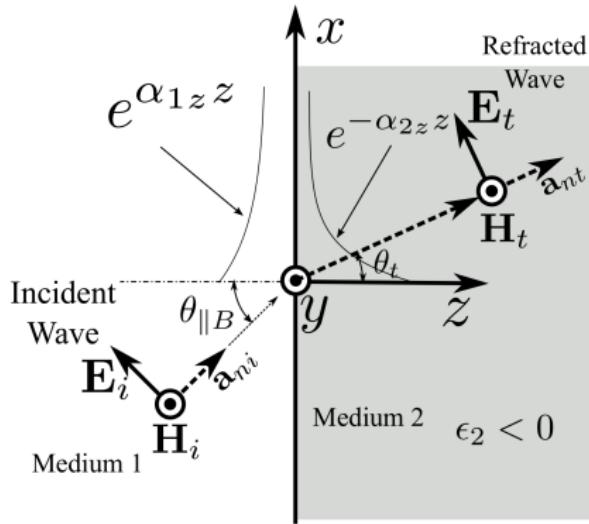
where  $\omega_p = \sqrt{\frac{Ne^2}{m^*\epsilon_0}}$  is the plasma oscillation frequency, and  $\omega_d$  is the damping oscillation frequency [Link].



Metal	plasma [eV/cm-1/PHz]	damping [meV/cm-1/THz]	source
Ag	9.6*/77430/2.321	22.8*/183.9/5.513	<a href="#">Blaber</a>
	9.013/72700*/2.18	18/145.2*/4.353	<a href="#">Ordal</a>
	9.04*/72920/2.186	21.25*/171.4/5.139	<a href="#">Zeman</a>
Al	8.6*/69370/2.08	45*/363/10.88	<a href="#">Hooper</a>
	15.3*/123000/3.7	598.4*/48.27/144.7	<a href="#">Blaber</a>
	14.75/119000*/3.57	81.8/660*/19.79	<a href="#">Ordal</a>
Au	12.04*/97110/2.911	128.7*/1038/31.12	<a href="#">Zeman</a>
	8.55*/69000/2.068	18.4*/148.4/4.449	<a href="#">Blaber</a>
	9.026/72800*/2.183	26.7/215*/6.46	<a href="#">Ordal</a>
Cu	8.89*/71710/2.15	70.88*/571.7/17.14	<a href="#">Zeman</a>
	9*/72590/2.176	70*/564.6/17.71	<a href="#">Berciaud</a>

[Link]

# Surface Wave on Plasma Dielectric Boundary



- Medium 1 ( $z < 0$ ),

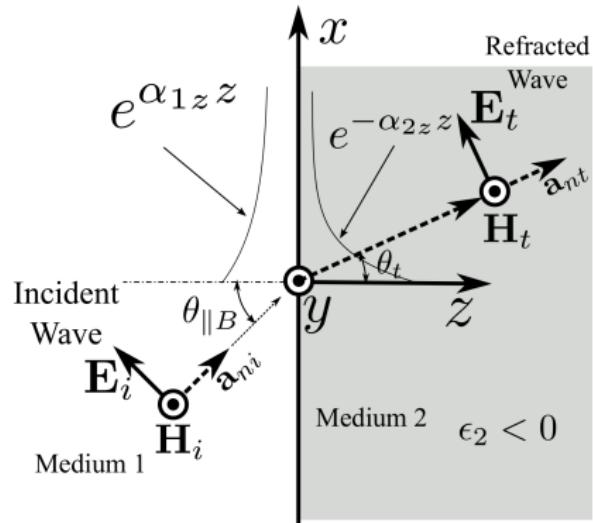
$$\begin{aligned}\mathbf{H}_1 &= \frac{E_{0i}}{\eta_1} \mathbf{a}_y e^{-j\beta_1(z \cos \theta_{B\parallel} + x \sin \theta_{B\parallel})} \\ &= \frac{E_{0i}}{\eta_1} \mathbf{a}_y e^{\alpha_1 z} e^{-j\beta_x x}\end{aligned}$$

- Medium 2 ( $z > 0$ ),

$$\begin{aligned}\mathbf{H}_2 &= \frac{E_{0t}}{\eta_2} \mathbf{a}_y e^{-j\beta_2(z \cos \theta_t + x \sin \theta_t)} \\ &= \frac{E_{0t}}{\eta_2} \mathbf{a}_y e^{-\alpha_2 z} e^{-j\beta_x x}\end{aligned}$$

# Surface Wave on Plasma Dielectric Boundary

Medium 1 ( $z < 0$ ),



$$\mathbf{H}_1 = \frac{E_{0i}}{\eta_1} \mathbf{a}_y e^{\alpha_{1z} z} e^{-j\beta_x x}$$

Medium 2 ( $z > 0$ ),

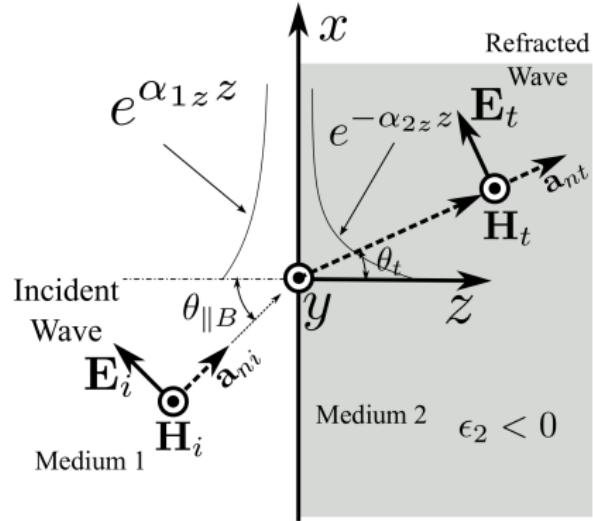
$$\mathbf{H}_2 = \frac{E_{0t}}{\eta_2} \mathbf{a}_y e^{-\alpha_{2z} z} e^{-j\beta_x x}$$

$$\beta_x = \beta_1 \sin \theta_{B\parallel} = \beta_2 \sin \theta_t = \frac{\beta_1}{\sqrt{1 + \epsilon_1/\epsilon_2}}$$

$$\alpha_{1z} = -j\beta_1 \cos \theta_{B\parallel} = \beta_1 \sqrt{\sin^2 \theta_{B\parallel} - 1} = \beta_1 \sqrt{\frac{-\epsilon_1/\epsilon_2}{1 + \epsilon_1/\epsilon_2}}$$

$$\alpha_{2z} = j\beta_2 \cos \theta_t = \sqrt{\beta_2^2 \sin^2 \theta_t - \beta_2^2} = \sqrt{\frac{\beta_1^2}{1 + \epsilon_1/\epsilon_2} - \beta_2^2}$$

# Surface Wave on Plasma Dielectric Boundary



$$\mathbf{H}_1 = \frac{E_{0i}}{\eta_1} \mathbf{a}_y e^{\alpha_1 z z} e^{-j\beta_x x}$$

$$\mathbf{H}_2 = \frac{E_{0t}}{\eta_2} \mathbf{a}_y e^{-\alpha_2 z z} e^{-j\beta_x x}$$

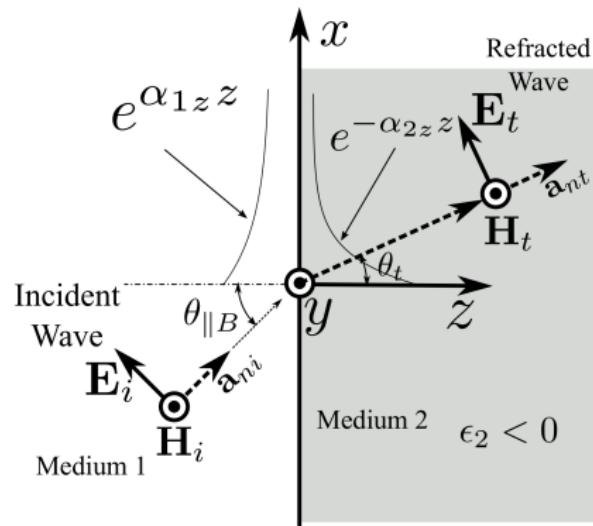
$$\beta_x = \beta_1 / \sqrt{1 + \epsilon_1 / \epsilon_2}$$

$$\alpha_{1z} = \beta_1 \sqrt{(-\epsilon_1 / \epsilon_2) / (1 + \epsilon_1 / \epsilon_2)}$$

$$\alpha_{2z} = j\beta_2 \cos \theta_t = \sqrt{\beta_2^2 \sin^2 \theta_t - \beta_2^2} = \sqrt{\frac{\beta_1^2}{1 + \epsilon_1 / \epsilon_2} - \beta_2^2}$$

$$\alpha_{2z} = \beta_1 \sqrt{\frac{1}{1 + \epsilon_1 / \epsilon_2} - \epsilon_2 / \epsilon_1} = \beta_1 \sqrt{\frac{-\epsilon_2 / \epsilon_1}{1 + \epsilon_1 / \epsilon_2}}$$

# Surface Wave on Plasma Dielectric Boundary



$$\mathbf{H}_1 = \frac{E_{0i}}{\eta_1} \mathbf{a}_y e^{\alpha_1 z} e^{-j\beta_x x}$$

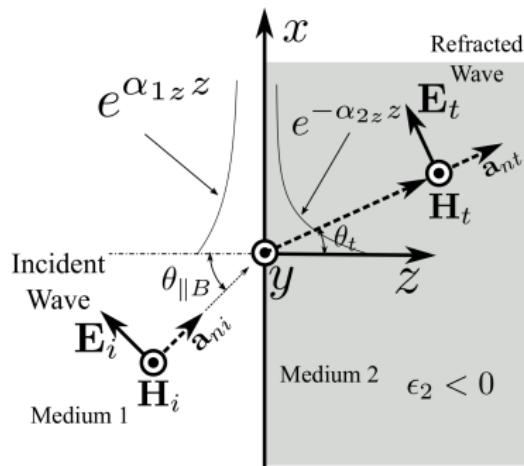
$$\mathbf{H}_2 = \frac{E_{0t}}{\eta_2} \mathbf{a}_y e^{-\alpha_2 z} e^{-j\beta_x x}$$

$$\beta_x = \beta_1 / \sqrt{1 + \epsilon_1 / \epsilon_2}$$

$$\alpha_{1z} = \beta_1 \sqrt{(-\epsilon_1 / \epsilon_2) / (1 + \epsilon_1 / \epsilon_2)}$$

$$\alpha_{2z} = \beta_1 \sqrt{\frac{-\epsilon_2 / \epsilon_1}{1 + \epsilon_1 / \epsilon_2}}$$

# Surface Wave on Plasma Dielectric Boundary



$$\beta_x = \beta_1 / \sqrt{1 + \epsilon_1 / \epsilon_2}$$

$$\alpha_{1z} = \beta_1 \sqrt{(-\epsilon_1 / \epsilon_2) / (1 + \epsilon_1 / \epsilon_2)}$$

$$\alpha_{2z} = \beta_1 \sqrt{\frac{-\epsilon_2 / \epsilon_1}{1 + \epsilon_1 / \epsilon_2}}$$

- For air-plasma interface  $\epsilon_1 = \epsilon_0$ , and

$$\epsilon_2 = \epsilon_0 \left( 1 - \frac{\omega_p^2}{\omega^2} \right), \quad \beta_1 = k_0$$

$$\frac{\beta_x}{k_0} = \sqrt{\frac{\omega_p^2 / \omega^2 - 1}{\omega_p^2 / \omega^2 - 2}},$$

$$\frac{\alpha_{1z}}{k_0} = \frac{1}{\sqrt{\omega_p^2 / \omega^2 - 2}}, \quad \frac{\alpha_{2z}}{k_0} = \frac{|\omega_p^2 / \omega^2 - 1|}{\sqrt{\omega_p^2 / \omega^2 - 2}}$$

# Surface Wave on Plasma Dielectric Boundary

$$\frac{\beta_x}{k_0} = \sqrt{\frac{\omega_p^2/\omega^2 - 1}{\omega_p^2/\omega^2 - 2}},$$

$$\frac{\alpha_{1z}}{k_0} = \frac{1}{\sqrt{\omega_p^2/\omega^2 - 2}}, \quad \frac{\alpha_{2z}}{k_0} = \frac{|\omega_p^2/\omega^2 - 1|}{\sqrt{\omega_p^2/\omega^2 - 2}}$$

