Lecture 3

High Power Microwave Sources
EEC746

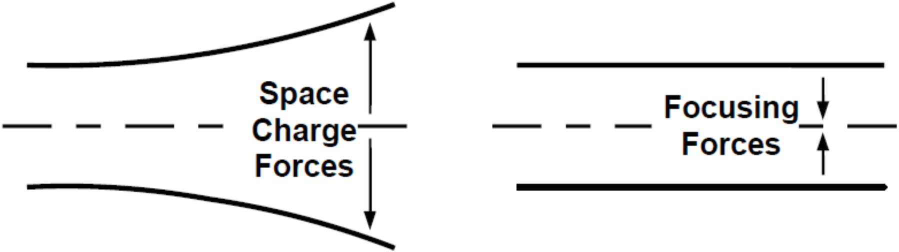
Tamer Abuelfadl

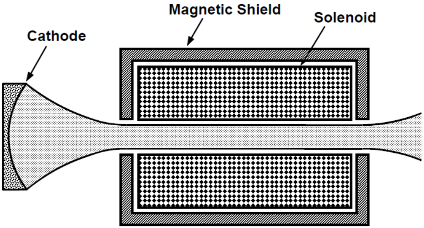
Electronics and Electrical Communications Department
Faculty of Engineering
Cairo University

Electron Dynamics

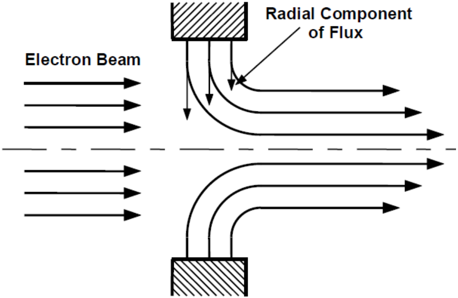
1. Overview of Uniform Field Focusing
   - Brillouin Flow
   - Scalloping
   - Confined (Immersed) Flow
2. Uniform-Field Focusing and Laminar Flow
   - The Beam Equation
   - Confined (Immersed) Flow

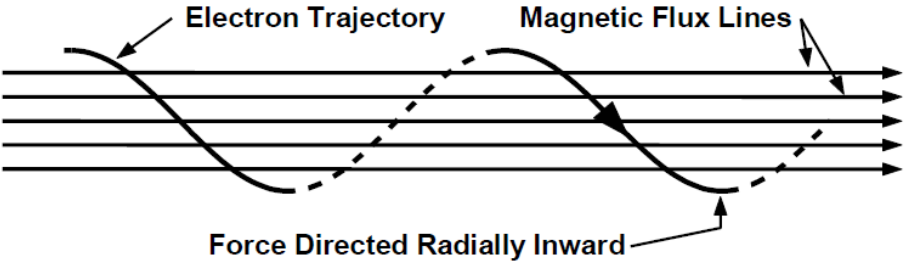
Electron Dynamics





Electron Dynamics

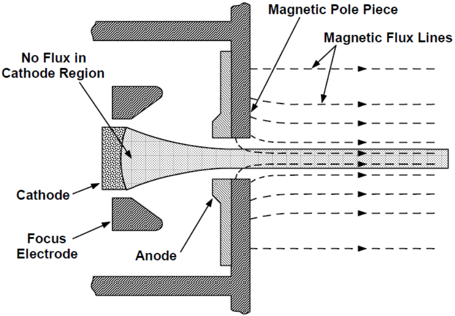


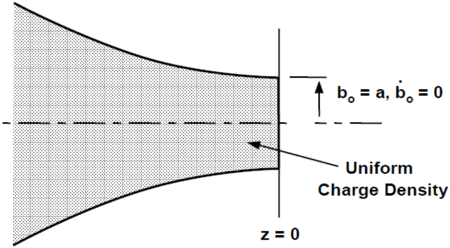


Outline

1. Overview of Uniform Field Focusing
   - Brillouin Flow
   - Scalloping
   - Confined (Immersed) Flow
2. Uniform-Field Focusing and Laminar Flow
   - The Beam Equation
   - Confined (Immersed) Flow

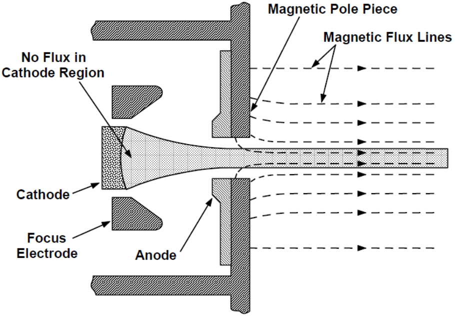
Brillouin Flow





Brillouin Flow

$$F_r = m\left(\ddot{r} - r\dot{\theta}^2\right) = -eE_r - er\dot{\theta}B_z$$



On the beam envelope $r = b$,

$$E_r = \frac{-I}{2\pi\varepsilon_0 u_0 b}$$

$$\ddot{b} - b\dot{\theta}^2 = \frac{\eta I}{2\pi\varepsilon_0 u_0 b} - \eta b\dot{\theta}B_z$$

According to Busch's theorem, $\dot{\theta} \approx \frac{\eta}{2}\left(B_z - \frac{r_0^2}{r^2}B_{z0}\right)$

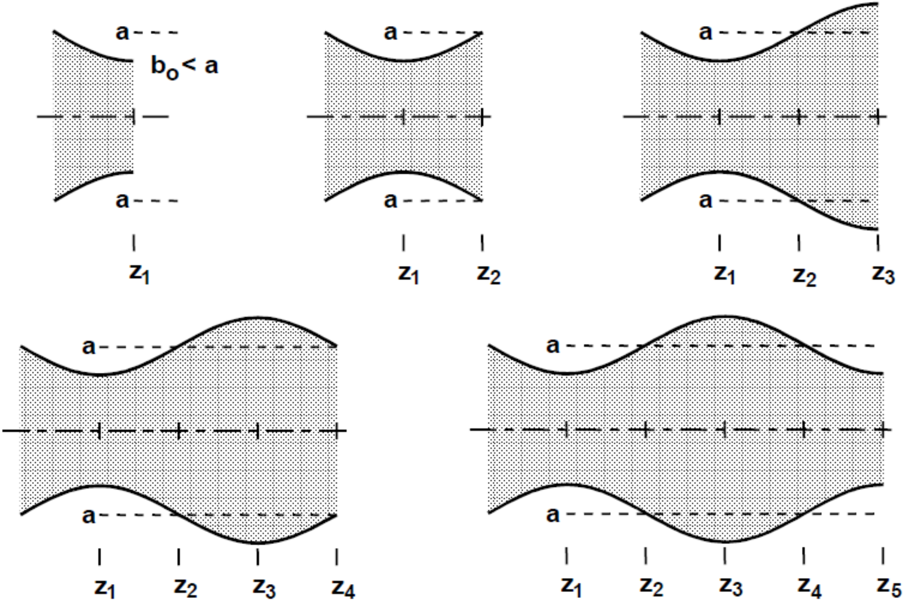
As in the cathode region $B_{z0} = 0, \dot{\theta} = \frac{\eta}{2}B_z = \omega_L \equiv$ Larmer frequency

$$\ddot{b} + \omega_L^2 b = \frac{\eta I}{2\pi\varepsilon_0 u_0 b}$$

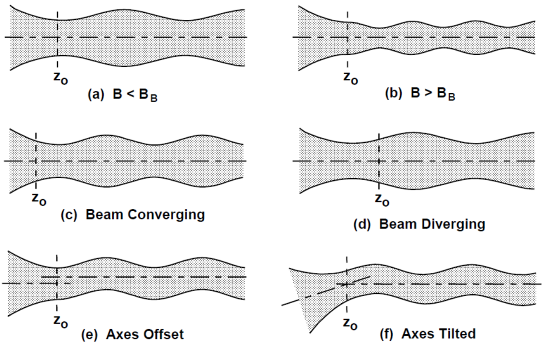
For Brillouin flow the equilibrium beam radius ($\ddot{b} = 0$), $a = \frac{1}{B_z}\sqrt{\frac{2I}{\pi\varepsilon_0 \eta u_0 b}}$.

Outline

Scalloping



1. Flux density is too small.
2. Flux density is too large.
3. Beam is converging at the entrance.
4. Beam is diverging at the entrance.
5. Beam axis is offset from the magnetic field axis.
6. Beam axis is tilted with respect to the magnetic field axis.



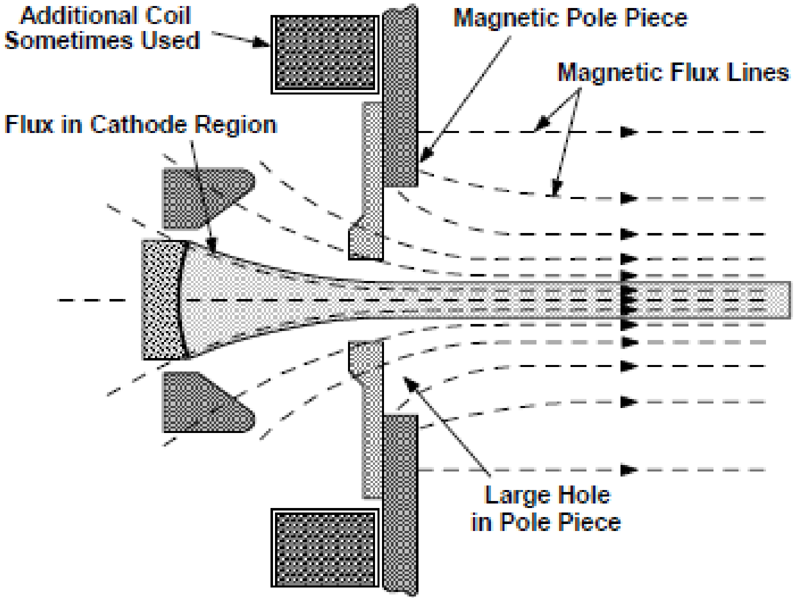
Brillioin Flow Pros and Cons

- Magnetic field required is the lower for any other focusing system.
- Beam is extremely sensitive to misalignment and perturbation.
- Brillioin flow can be very nearly achieved under laboratory conditions. In practice, the magnetic focusing field that is used, even when the cathode is shielded, is greater than the Brillouin value. Reasons for this include transverse electron velocities (from thermal and other effects in the gun) and the increase in beam size that results from RF modulation.

Outline

1. Overview of Uniform Field Focusing
   - Brillouin Flow
   - Scalloping
   - Confined (Immersed) Flow
2. Uniform-Field Focusing and Laminar Flow
   - The Beam Equation
   - Confined (Immersed) Flow

Confined (Immersed) Flow



Outline

$$\ddot{r} - r\dot{\theta}^2 = -\eta\left(E_r + Br\dot{\theta}\right),$$

where $\dot{\theta}$ is given by Busch's Theorem,

$$\dot{\theta} = \frac{\eta}{2}\left(B - B_c\frac{r_c^2}{r^2}\right)$$

The electric field on the beam boundary $r = b$,

$$E_r(b) = \frac{-I}{2\pi b\varepsilon_0 u_0},$$

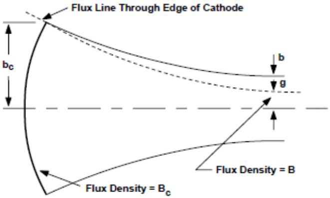
The envelope equation is given by,

$$\ddot{b} - b\frac{\eta^2}{4}\left(B - B_c\frac{b_c^2}{b^2}\right)^2 = -\eta\left[\frac{-I}{2\pi b\varepsilon_0 u_0} + Bb\frac{\eta}{2}\left(B - B_c\frac{b_c^2}{b^2}\right)\right]$$

$$\ddot{b} + b\omega_L^2\left(1 - \frac{B_c^2}{B^2}\frac{b_c^4}{b^4}\right) - \frac{\eta I}{2\pi b\varepsilon_0 u_0} = 0$$

The Beam Equation

$$\ddot{b} + b\omega_L^2 \left(1 - \frac{B_c^2}{B^2}\frac{b_c^4}{b^4}\right) - \frac{\eta I}{2\pi b \varepsilon_0 u_0} = 0$$



$$\pi b_c^2 B_c = \pi g^2 B \qquad \Longrightarrow \qquad \frac{B_c}{B}\frac{b_c^2}{b^2} = \frac{g^2}{b^2}$$

Using the Brillouin radius $a = \frac{1}{\omega_L}\left(\frac{\eta I}{2\pi\varepsilon_0 u_0}\right)^{1/2}$,

$$\ddot{b} + \omega_L^2 \left[b\left(1 - \frac{g^4}{b^4}\right) - \frac{a^2}{b} \right] = 0$$

Normalizing the radii, $R = \frac{b}{a}$, and $R_g = \frac{g}{a}$,

$$\ddot{R} + \omega_L^2 \left[R\left(1 - \frac{R_g^4}{R^4}\right) - \frac{1}{R} \right] = 0$$

The Beam Equation

$$\ddot{R} + \omega_L^2 \left[R \left(1 - \frac{R_g^4}{R^4} \right) - \frac{1}{R} \right] = 0$$

Equilibrium radius R_e corresponds to $\ddot{R}_e = 0$, and is given by the following equation,

$$R_e \left(1 - \frac{R_g^4}{R_e^4} \right) - \frac{1}{R_e} = 0 \qquad \Longrightarrow \qquad R_e = \left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4R_g^4} \right]^{1/2}$$

The Beam Equation

$$\ddot{R} + \omega_L^2 \left[R \left(1 - \frac{R_g^4}{R^4} \right) - \frac{1}{R} \right] = 0$$

So the solution near the equilibrium $R = R_e (1 + \delta)$ can be linearized as follows,

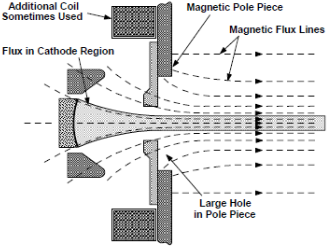
$$\ddot{\delta} + 2\Omega\omega_L^2 \delta = 0, \qquad \text{where } \Omega = 2 - \frac{1}{R_e^2}$$

$$\delta = A \cos\left(\sqrt{2\Omega}\omega_L t\right) + A \sin\left(\sqrt{2\Omega}\omega_L t\right)$$

$$R = R_e \left[1 + A \cos\left(\sqrt{2\Omega} \frac{\omega_L}{u_0} z \right) + A \sin\left(\sqrt{2\Omega} \frac{\omega_L}{u_0} z \right) \right]$$

Outline

Confined (Immersed) Flow



Absence of flux in cathode region

$$-b_0\,\dot\theta_B^2 = -\eta\left(E_r + B_B b_0\,\dot\theta_B\right),$$

$$\dot\theta_B = \frac{\eta}{2} B_B$$

Presence of flux in cathode region

$$-b_0\,\dot\theta^2 = -\eta\left(E_r + B b_0\,\dot\theta\right)$$

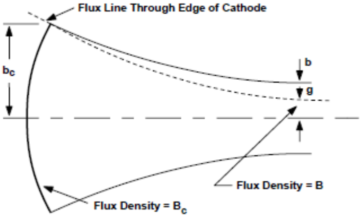
$$\dot\theta = \frac{\eta}{2}\left(B - B_c\frac{b_c^2}{b_0^2}\right)$$

$$b_0\,\dot\theta_B\left(\eta B_B - \dot\theta_B\right) = b_0\,\dot\theta\left(\eta B - \dot\theta\right)$$

$$B^2 = B_B^2 + B_c^2\frac{b_c^2}{b_0^2}$$

Confined (Immersed) Flow

$$B^2 = B_B^2 + B_c^2 \frac{b_c^2}{b_0^2}$$



Definition

The confinement factor m is defined as,

$$B = mB_B$$

$$b_c^2 B_c = g^2 B = g^2 m B_B, \quad \rightarrow g = \left(1 - \frac{1}{m^2}\right)^{1/4} b_0, \quad \rightarrow R_g = \left(1 - \frac{1}{m^2}\right)^{1/4} R_e$$

$$\because \quad R_e \left(1 - \frac{R_g^4}{R_e^4}\right) - \frac{1}{R_e} = 0, \qquad \therefore R_e = m,$$

$$R_g = \left[m^2 \left(m^2 - 1\right)\right]^{1/4}$$