Sheet 2

1. (Cheng 8-4) For a harmonic uniform plane wave propagating in a simple medium, both $\mathbf{E}$ and $\mathbf{H}$ vary in accordance with the factor $\exp(-j\mathbf{k}\cdot\mathbf{R})$. Show that the four Maswell's equations for uniform plane wave in a source-free region reduce to the following:

$$\mathbf{k}\times\mathbf{E}=\omega\mu\mathbf{H}$$

$$\mathbf{k}\times\mathbf{H}=-\omega\epsilon\mathbf{E}$$

$$\mathbf{k}\cdot\mathbf{E}=0,\qquad \mathbf{k}\cdot\mathbf{H}=0$$

2. (Cheng 8-5) The instantaneous expression for the magnetic field intensity of a uniform plane wave propagating in the $+y$ direction in air is given by,

$$\mathbf{H}=\mathbf{a}_z 4\times10^{-6}\cos\left(10^7\pi t-k_0 y+\frac{\pi}{4}\right)\qquad \text{A/m}$$

   (a) Determine k_0 and the location where $H_z$vanishes at $t=3$ ms.

   (b) Write the instantaneous expression for $\mathbf{E}$.

3. (Cheng 8-6) The $\mathbf{E}$-field of a uniform plane wave propagating in a dielectric medium is given by,

$$\mathbf{E}(z,t)=\mathbf{a}_x 2\cos\left(10^8 t-z/\sqrt{3}\right)-\mathbf{a}_y 2\sin\left(10^8 t-z/\sqrt{3}\right)\qquad \text{V/m}$$

   (a) Determine the frequency and wavelength of the wave.

   (b) What is the dielectric constant of the medium?

   (c) Describe the polarization of the wave.

   (d) Find the corresponding $\mathbf{H}$-field.

4. (Cheng 8-9) Derive the following general expressions of the attenuation and phase constants for conducting media:

$$\alpha=\omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2}\qquad \text{Np/m}$$

$$\beta=\omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2},\qquad \text{rad/m}$$

5. (Cheng 8-10) Determine and compare the intrinsic impedance, attenuation constant (in both Np/m and dB/m), and skin depth of copper [$\sigma_{\text{cu}}=5.8\times10^7$ S/m], silver [$\sigma_{\text{ag}}=6.15\times10^7$ S/m], and brass [$\sigma_{\text{br}}=1.59\times10^7$ S/m] at the following frequencies: (a) 60 Hz, (b) 1 MHz, and (c) 1 GHz.

6. (Cheng 8-12) The magnetic field intensity of a linearly polarized uniform plane wave propagating in the $+y$-direction in seawater [$\epsilon_r = 80$, $\mu_r = 1$, $\sigma = 4$ S/m] is

$$\mathbf{H} = \mathbf{a}_x 0.1 \sin\left(10^{10}\pi t - \pi/3\right) \qquad \text{A/m}$$

at $y = 0$.

   (a) Determine the attenuation constant, the phase constant, the intrinsic impedance, the phase velocity, the wavelength, and the skin depth.

   (b) Find the location at which the amplitude of $\mathbf{H}$ is 0.01 A/m.

   (c) Write the expressions for $\mathbf{E}(y,t)$ and $\mathbf{H}(y,t)$ at $y = 0.5$ m as functions of t.

7. (Cheng 8-13) Given that the skin depth for graphite at 100 MHz is 0.16 mm, determine

   (a) the conductivity of graphite, and

   (b) the distance that at 1 GHz wave travels in graphite such that its field intensity is reduced to 30 dB.