EXERCISE V: ANTENNA ARRAYS

Problem (1)

Find the space factor in the following cases (use conventional coordinate system):

1. Two equal in-phase sources at (0, 0, 0) and (*d*, 0, 0).
2. Two equal in-phase sources at (0, 0, 0) and (*d*, *d*, 0).
3. Four equal in-phase sources at (0, 0, 0), (*d*, 0, 0), (0, *d*, 0) and (*d*, *d*, 0).
4. Three in-phase sources at (0, 0, 0), (0, 0, *d*) and (0, 0, 2*d*) with ratio of magnitudes 1:2:1.

Problem (2)

Show that the array factor of a binomial array with four elements excited in phase with relative amplitudes 1:3:3:1 is given by:

$$S(\theta) = \cos^3\left(\frac{kd}{2}\cos(\theta)\right), \qquad d \text{ is the inter-element spacing}$$

Hence, draw the pattern and determine the 3-dB beam width for $d=\lambda/2$.

Problem (3)

1. Show that for long linear arrays of isotropic sources the beam width between the first nulls is approximately given by:

$$BWBFN = \begin{cases} \dfrac{2\lambda}{Nd}, & \text{in case of broadside array} \\ 2\sqrt{\dfrac{2\lambda}{Nd}}, & \text{in case of end-fire array} \end{cases}$$

where *N* is the total number of elements

d is the inter-element spacing

2. For a large broadside array, show that the amplitude of the m^{th} side-lobe relative to the main beam is approximately given by:

$$\frac{1}{N\sin\left(\frac{\pi}{2N}(2m+1)\right)}$$

Problem (4)

1. Use the directional pattern of a thin center-fed $\lambda/2$ dipole to obtain the pattern of a center-fed λ dipole.
2. Show how the radiation pattern of a straight travelling wave wire antenna of arbitrary length can be obtained using the principle of pattern multiplication.

Problem (5)

Determine the space factor of a linear end-fire array of eight elements spaced $\lambda/4$ apart, giving directions and relative magnitudes of side-lobes. Sketch the pattern showing the positions of nulls.

Problem (6) (*Jan. 1997*)
A two-element antenna array consists of thin half-wave dipoles on the *Z*-axis at z=0 and z=d. The dipoles are parallel to the *X*-axis with feed currents I_0 and $I_0 e^{-j\alpha d}$.
Find α and d in order to have zero radiation in the negative *Z* direction and maximum in the positive *Z* direction. Sketch the directional patterns in the principal planes.

Problem (7)
Four vertical antennas, parallel to *Z*-axis, are equally spaced along the *X*-axis, and excited with equal currents. If the inter-element spacing is 0.8λ, determine the radiation pattern in the *XY*-plane.

Problem (8)
1. A linear array of N=50 equally spaced elements with $d=\lambda/2$ is excited for end-fire operation. If the elements are along the *Z*-axis, give an expression for the space factor *S* and find its first two zeros.
2. If the array consists of equal dipoles parallel to the *X*-axis find the radiation pattern in the *XY* and *XZ* planes. What is the difference for N=49?

Problem (9)
A linear array 80 wavelengths long has equally spaced elements with $\lambda/2$ inter-element spacing. The elements are excited with equal in-phase currents. Assuming isotropic elements, determine the directions and widths of the principal maximum as well as the side-lobe level in the two cases:
1. All elements (N=161) in operation.
2. Elements 1, 5, 9,......, 157, 161 in operation.

Problem (10)
Consider a two-element array with $d=2\lambda$, fed with in-phase, equal magnitude currents. Sketch, <u>without</u> formulation, the space factor pattern, showing nulls and maxima. Comment on the values of lobe maxima.
The inter-element separation is now filled with three additional elements to form a 5 element broadside array. Obtain an expression for the space-factor (*S*) and determine the zeros as well as the side-lobe level (*SLL*).

Problem (11)
A linear array consists of 60 parallel coplanar equally spaced $\lambda/2$ dipoles parallel to the *Z*-axis, operating at 1 GHz with equal magnitude current excitation. The array is placed $\lambda/4$ above and parallel to a perfectly conducting screen in the *XZ* plane with inter-element spacing of 0.4λ.
1. Determine, for broadside operation, the radiation pattern in *XY* and *XZ* plane.
2. Give the beam width between nulls and the directions and relative levels of side-lobes with beam scanning 60° off broadside in the *XY* plane, keeping the exciting current magnitude unchanged. Give the change in maximum radiation intensity relative to its value at broadside.