Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Total number of points in this Exam 24 points.

Name : ______________________ Student ID#: ______________________

Question 1 ()

The radiation intensity of an antenna is given by,

$$U(\theta, \phi) = \cos^4\theta \cos^2\phi + \sin^2\phi,$$

for $0 \le \theta \le \pi/2$ and $0 \le \phi \le 2\pi$ (i.e., in the upper half-space). It is zero in the lower half-space.

(a) (4 points) Find the exact directivity (dimensionless and in dB) .

(b) (2 points) Find the beam solid angle.

(c) (4 points) Sketch the radiation pattern in elevation $x - z$ plan and find half-power beamwidth (in degrees).

(d) (2 points) Sketch the radiation pattern in elevation $y - z$ plan.

Question 2 (4 points)

A half-wave dipole is radiating into free-space. The coordinate system is defined so that the origin is at the center of the dipole and the z-axis is aligned with the dipole. Input power to the dipole is 100 W. Assuming an overall efficiency of 50%, find the power density (in $\mathrm{W/m^2}$) at $r = 500$ m, $\theta = 60^\circ$, $\phi = 0^\circ$.

Question 3 ()

A circular loop, of loop radius $\lambda/30$ and wire radius $\lambda/1000$, is used as a transmitting antenna in a back-pack radio communication system at 10 MHz. The wire of the loop is made of copper with a conductivity of 5.7×10^7 ℧/m. Assuming the antenna is radiating in free space, determine the

(a) (3 points) radiation resistance of the loop;

(b) (3 points) loss resistance of the loop (assume that its value is the same as if the wire were straight);

(c) (2 points) radiation efficiency.

Formula Sheet

- Radiation electric field from dipole of length l along z-axis,

$$\mathbf{E} = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right] \hat{\mathbf{a}}_\theta$$

- For infinitesimal dipole of length l ($l \ll \lambda$) along direction $\hat{\mathbf{n}}$,
  - Radiation electric field

$$\mathbf{E} = -j\eta \frac{klI_0 e^{-jkr}}{4\pi r} \left[\hat{\mathbf{n}} - (\hat{\mathbf{n}} \cdot \hat{\mathbf{a}}_r)\,\hat{\mathbf{a}}_r\right]$$

  - Radiation Resistance

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2.$$

- Radiation electric field from a small loop of radius a ($2\pi a < \lambda/10$) with its axis oriented along direction $\hat{\mathbf{n}}$,

$$\mathbf{E} = \eta \frac{k^2 a^2 I_0 e^{-jkr}}{4r} \hat{\mathbf{n}} \times \hat{\mathbf{a}}_r$$

- Directivity of $\lambda/2$ dipole is 1.643 and radiation resistance is 73 Ω.
- Conductor surface resistance $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}}$, where $\mu_0 = 4\pi \times 10^{-7}$ H/m.
- Matrix transformation of any vector from Cartesian to spherical coordinates,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

- $$\int e^{\alpha x}\sin(\beta x + \gamma) = \frac{e^{\alpha x}}{\alpha^2 + \beta^2}\left[\alpha\sin(\beta x + \gamma) - \beta\cos(\beta x + \gamma)\right]$$

- Angle ψ between the two directions $(\theta,\ \phi)$ and $(\theta',\ \phi')$,

$$\cos\psi = \sin\theta\sin\theta'\cos(\phi - \phi') + \cos\theta\cos\theta'$$

- Beam solid angle Ω_A,

$$\Omega_A = \frac{\oiint_{4\pi} U d\Omega}{U_{max}}$$

- Aperture Efficiency ϵ_{ap} is,

$$\epsilon_{ap} = \frac{\left|\iint_{A_p} \mathbf{E}_a ds'\right|^2}{A_p \iint_{A_p} |\mathbf{E}_a|^2 ds'}$$

- Antenna Vector Effective Length $\boldsymbol{\ell}$,

$$\mathbf{E} = j\eta \frac{kI_{in}}{4\pi r} \boldsymbol{\ell} e^{-jkr}$$

- Radiation fields due to electric and magnetic surface currents,

$$E_\theta = -j\omega\left(A_\theta + j\eta F_\phi\right), \qquad E_\phi = -j\omega\left(A_\phi - j\eta F_\theta\right)$$

$$\mathbf{A} = \frac{\mu e^{-jkr}}{4\pi r}\mathbf{N}, \qquad \mathbf{F} = \frac{\epsilon e^{-jkr}}{4\pi r}\mathbf{L}$$

$$\mathbf{N} = \iint_S \mathbf{J}_s e^{jkr'\cos\psi} ds' = \iint_S \mathbf{J}_s e^{j\mathbf{k}\cdot\mathbf{r}'} ds', \qquad \mathbf{L} = \iint_S \mathbf{M}_s e^{jkr'\cos\psi} ds' = \iint_S \mathbf{M}_s e^{j\mathbf{k}\cdot\mathbf{r}'} ds',$$

where $\quad \mathbf{k}\cdot\mathbf{r}' = k\left(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta\right)$

- Electric current on a perfect electric conductor: $\quad \mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}$
- Magnetic current on a perfect magnetic conductor: $\quad \mathbf{M}_s = -\hat{\mathbf{n}} \times \mathbf{E}$