Faculty of Engineering
Cairo University
Electronics and Electrical Engineering Dept.

Electromagnetic II (ELC N405)
2014

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Total number of points in this Exam 24 points.

Name : ______________________ Student ID#: ______________________

Question 1 ()

The radiation intensity of an antenna is given by,

$$U(\theta, \phi) = \cos^4\theta \cos^2\phi + \sin^2\phi,$$

for $0 \leq \theta \leq \pi/2$ and $0 \leq \phi \leq 2\pi$ (i.e., in the upper half-space). It is zero in the lower half-space.

(a) (4 points) Find the exact directivity (dimensionless and in dB) .

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi) \sin\theta \, d\theta \, d\phi$$

$$= \pi \int_{\theta=0}^{\pi/2} [\cos^4\theta + 1] \sin\theta \, d\theta = \pi \left[-\frac{\cos^5\theta}{5} - \cos\theta \right]_0^{\pi/2}$$

$$= \frac{6\pi}{5}$$

$$\boxed{D_{max} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \times 1}{6\pi/5} = \frac{10}{3}}$$

$$\boxed{D_{max}(dB) = 5.23 \text{ dB}}$$

4

(b) (2 points) Find the beam solid angle.

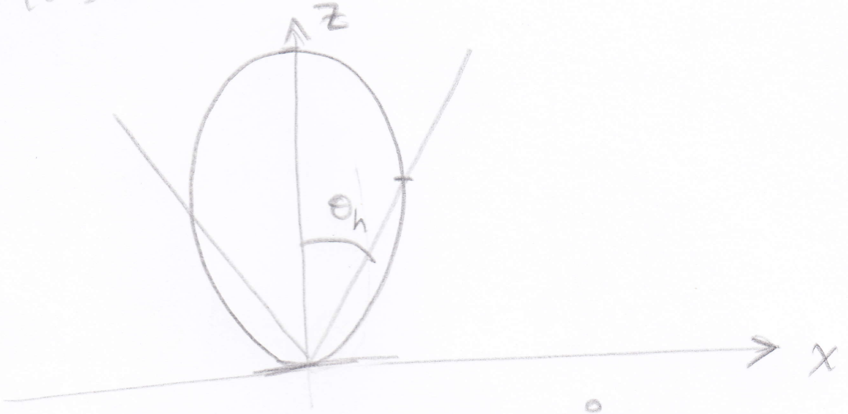
$$\boxed{\Omega_A = \frac{4\pi}{D_{max}} = 4\pi \times \frac{3}{10} = 1.2\pi = 3.77 \text{ str}}$$

2

(c) (4 points) Sketch the radiation pattern in elevation $x - z$ plane and find half-power beamwidth (in degrees).

$x-z$ plane $\underline{\phi = 0}$

$U(\theta) = \cos^4\theta$



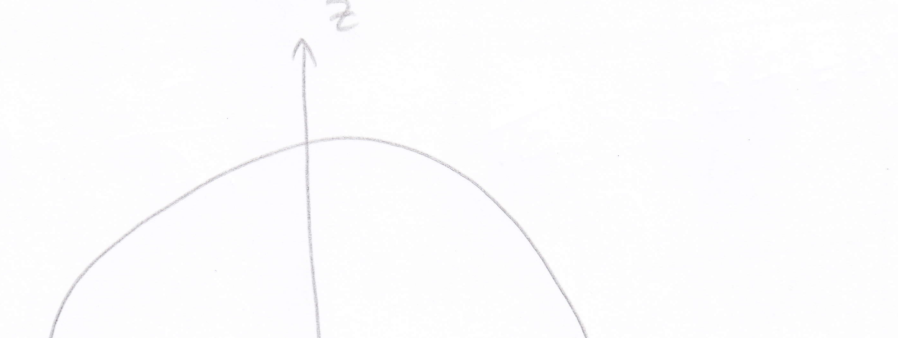
$\cos^4\theta_h = \frac{1}{2} \Rightarrow \theta_h = 32.77^\circ$

$\boxed{HPBW = 2\theta_h = 65.53^\circ}$

(d) (2 points) Sketch the radiation pattern in elevation $y - z$ plane.

$y-z$ plane $\phi = 90^\circ$

$U(\theta)=1$



Question 2 (4 points)

A half-wave dipole is radiating into free-space. The coordinate system is defined so that the origin is at the center of the dipole and the z-axis is aligned with the dipole. Input power to the dipole is 100 W. Assuming an overall efficiency of 50%, find the power density (in W/m^2) at $r = 500$ m, $\theta = 60°$, $\phi = 0°$.

$P_{in} = 100$ W $\quad \eta = 50\%$

$$P_{rad} = P_{in}\,\eta = 50\text{ W}$$

$$W_{rad} = \frac{P_{rad}}{4\pi r^2} D(\theta = 60°, \phi = 0°)$$

$\because D_{max} = 1.64$

Dependance of the E field $\dfrac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$

$$D(\theta) = 1.64 \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

$$D(\theta = 60°) = 1.64 \left[\frac{\cos(\frac{\pi}{2}\times\cos 60°)}{\sin 60°}\right]^2$$

$$= 1.64 \times \left[\frac{1/\sqrt{2}}{\sqrt{3}/2}\right]^2 = 1.64 \times \frac{2}{3}$$

$$D(\theta = 60°) = 1.093$$

$$\boxed{W_{rad} = \frac{50}{4\pi \times (500)^2} \times 1.093 = 1.74 \times 10^{-5}\ \text{W/m}^2}$$

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19/10

Question 3

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A circular loop, of loop radius $\lambda/30$ and wire radius $\lambda/1000$, is used as a transmitting antenna in a back-pack radio communication system at 10 MHz. The wire of the loop is made of copper with a conductivity of 5.7×10^7 ℧/m. Assuming the antenna is radiating in free space, determine the

(a) (3 points) radiation resistance of the loop;

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{|E|^2}{2\eta} r^2 \sin\theta \, d\theta \, d\phi = \eta \frac{(ka)^4 |I_0|^2}{\cancel{32}\ 16} \times \cancel{2\pi} \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta$$

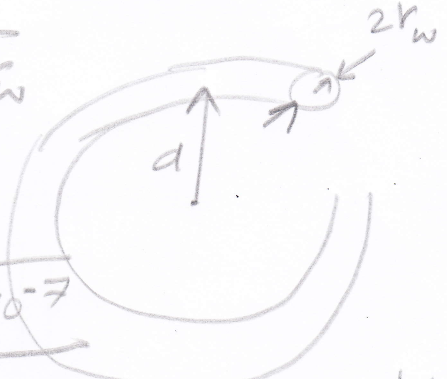
$$P_{rad} = \eta \frac{(ka)^4 |I_0|^2}{16} \pi \left[-\cos\theta + \frac{\cos^3\theta}{3} \right]_{\theta=0}^{\pi}$$

$$= \eta \frac{(ka)^4 |I_0|^2}{\cancel{16}\ 4} \times \pi \times \frac{4}{3} = 10\pi^2 (ka)^4 |I_0|^2$$

$$\boxed{R_{rad} = 20\pi^2 (ka)^4 = 0.38\,\Omega}$$

(b) (3 points) loss resistance of the loop (assume that its value is the same as if the wire were straight);

$$R_{Loss} = R_s \frac{L}{2\pi r_w} = R_s \frac{2\pi a}{2\pi r_w}$$



$$R_{Loss} = R_s \frac{a}{r_w}$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\frac{\pi \times 10 \times 10^6 \times 4\pi \times 10^{-7}}{5.7 \times 10^7}}$$

$$= 8.322 \times 10^{-4}\,\Omega$$

$$R_{Loss} = 8.322 \times 10^{-4} \times \frac{1/30}{1/1000}$$

$$\boxed{R_{Loss} = 0.028\,\Omega}$$

(c) (2 points) radiation efficiency.

$$\eta_{rad} = \frac{R_{rad}}{R_{rad} + R_{Loss}} = \frac{0.38}{0.38 + 0.028}$$

$$\boxed{\eta_{rad} = 93\%}$$