EXERCISE III: ANTENNA TYPES AND ANALYSIS TECHNIQUES

Problem (1)

Compute the directivity of long wires having lengths 2λ and 2.5λ supporting a standing-wave current distribution. (Hint: use the values of the integrals provided in the attached Integral Formula).

Problem (2)

Show that for a uniformly illuminated rectangular aperture of dimensions $a \times b$, the maximum gain is given by:

$$G_0 = \frac{4\pi}{\lambda^2}(ab)$$

Problem (3)

Consider a rectangular aperture of dimensions $a = 30\lambda$ and $b = 15\lambda$ in the *XY* plane exists in a large conducting screen.

1. If the aperture is illuminated by a uniform TEM wave with $\mathbf{E} = E_0\hat{\mathbf{x}}, \mathbf{k} = k\hat{\mathbf{z}}$, determine:
   a. The radiated fields, the FNBW and the SLL in the principal planes.
   b. The gain.
   c. Total maximum directivity D_o (*dimensionless* and *in dB*).
2. Repeat (1) if the aperture is non-uniformly illuminated with electric field $\mathbf{E} = E_0 \cos\left(\frac{\pi x}{a}\right)\hat{\mathbf{x}}$ (i.e. the dominant mode of a rectangular wave guide). Hence, determine the illumination efficiency.

Integral Formula

$$R(x) = \int_0^{\pi} \frac{[\cos(x\cos\theta) - \cos x]^2}{\sin\theta} d\theta$$

$$R\left(\frac{\pi}{2}\right) = 1.21883$$

$$R(\pi) = 3.31813$$

$$R\left(\frac{3\pi}{2}\right) = 1.75824$$

$$R(2\pi) = 4.32724$$

$$R\left(\frac{5\pi}{2}\right) = 2.01277$$