Topic 5

Aperture Antennas

Tamer Abuelfadl

Electronics and Electrical Communications Department
Faculty of Engineering
Cairo University

Outline

Balanced Maxwell's Equations

Equations due to Electric Current,

$$\begin{aligned} \nabla \times \mathbf{E}_A &= -j\omega\mu\mathbf{H}_A \\ \nabla \times \mathbf{H}_A &= j\omega\varepsilon\mathbf{E}_A + \mathbf{J} \\ \nabla \cdot \mathbf{E}_A &= \rho_e/\varepsilon \\ \nabla \cdot \mathbf{H}_A &= 0 \end{aligned}$$

Equations due to Magnetic Current,

$$\begin{aligned} \nabla \times \mathbf{E}_F &= -j\omega\mu\mathbf{H}_F - \mathbf{M} \\ \nabla \times \mathbf{H}_F &= j\omega\varepsilon\mathbf{E}_F \\ \nabla \cdot \mathbf{E}_F &= 0 \\ \nabla \cdot \mathbf{H}_F &= \rho_m/\mu \end{aligned}$$

Balanced Maxwell's Equations due to Electric and Magnetic Current,

$$\begin{aligned} \nabla \times \mathbf{E} &= -j\omega\mu\mathbf{H} - \mathbf{M} \\ \nabla \times \mathbf{H} &= j\omega\varepsilon\mathbf{E} + \mathbf{J} \\ \nabla \cdot \mathbf{E} &= \rho_e/\varepsilon \\ \nabla \cdot \mathbf{H} &= \rho_m/\mu \end{aligned}$$

$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F, \qquad \mathbf{H} = \mathbf{H}_A + \mathbf{H}_F$$

Continuity Equations, $\nabla \cdot \mathbf{J} = -j\omega\rho_e, \qquad \nabla \cdot \mathbf{M} = -j\omega\rho_m$

The Vector Potential for an Electric Current Source

$$\begin{aligned} \nabla \times \mathbf{E}_A &= -j\omega\mu\mathbf{H}_A \\ \nabla \times \mathbf{H}_A &= j\omega\varepsilon\mathbf{E}_A + \mathbf{J} \\ \nabla \cdot \mathbf{E}_A &= \rho_e/\varepsilon \\ \nabla \cdot \mathbf{H}_A &= 0 \end{aligned}$$

$$\boxed{\mathbf{H}_A = \frac{1}{\mu}\nabla \times \mathbf{A}, \qquad \mathbf{E}_A = -\nabla\phi_e - j\omega\mathbf{A}}$$

If , $\phi_e = -\dfrac{1}{j\omega\mu\varepsilon}\nabla \cdot \mathbf{A}$, then $\boxed{\nabla^2\mathbf{A} + k^2\mathbf{A} = -\mu\mathbf{J}}$

$$\boxed{\mathbf{E}_A = -\nabla\phi_e - j\omega\mathbf{A} = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla \cdot \mathbf{A})}$$

$$\begin{aligned}
\nabla \times \mathbf{E}_F &= -j\omega\mu\mathbf{H}_F - \mathbf{M} \\
\nabla \times \mathbf{H}_F &= j\omega\varepsilon\mathbf{E}_F \\
\nabla \cdot \mathbf{E}_F &= 0 \\
\nabla \cdot \mathbf{H}_F &= \rho_m/\varepsilon
\end{aligned}$$

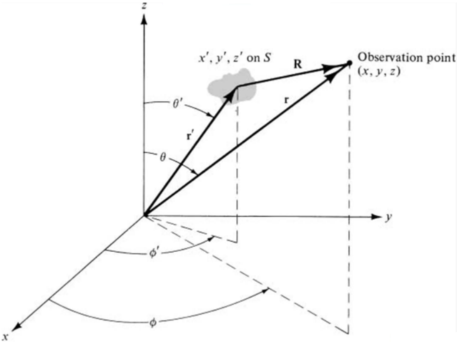
$$\boxed{\mathbf{E}_F = -\frac{1}{\varepsilon}\nabla \times \mathbf{F}, \qquad \mathbf{H}_F = -\nabla\phi_m - j\omega\mathbf{F}}$$

If , $\phi_m = -\dfrac{1}{j\omega\mu\varepsilon}\nabla \cdot \mathbf{F}$, then $\boxed{\nabla^2\mathbf{F} + k^2\mathbf{F} = -\varepsilon\mathbf{M}}$

$$\boxed{\mathbf{H}_F = -\nabla\phi_m - j\omega\mathbf{F} = -j\omega\mathbf{F} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla \cdot \mathbf{F})}$$

Balanced Maxwell's Equations due to Electric and Magnetic Current

$$
\begin{aligned}
\nabla \times \mathbf{E} &= -j\omega\mu\mathbf{H} - \mathbf{M} \\
\nabla \times \mathbf{H} &= j\omega\varepsilon\mathbf{E} + \mathbf{J} \\
\nabla \cdot \mathbf{E} &= \rho_e/\varepsilon \\
\nabla \cdot \mathbf{H} &= \rho_m/\mu
\end{aligned}
$$



$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F, \qquad \mathbf{H} = \mathbf{H}_A + \mathbf{H}_F$$

$$\mathbf{E} = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{A}) - \frac{1}{\varepsilon}\nabla\times\mathbf{F}$$

$$\mathbf{H} = -j\omega\mathbf{F} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{F}) + \frac{1}{\mu}\nabla\times\mathbf{A}$$

$$\nabla^2\mathbf{A} + k^2\mathbf{A} = -\mu\mathbf{J}$$

$$\nabla^2\mathbf{F} + k^2\mathbf{F} = -\varepsilon\mathbf{M}$$

$$\mathbf{A} = \frac{\mu}{4\pi}\iiint_V \mathbf{J}\frac{e^{-jkR}}{R}dv'$$

$$\mathbf{F} = \frac{\varepsilon}{4\pi}\iiint_V \mathbf{M}\frac{e^{-jkR}}{R}dv'$$

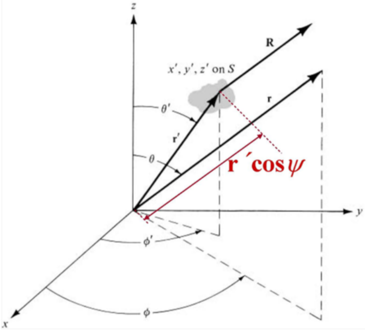
$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F, \qquad \mathbf{H} = \mathbf{H}_A + \mathbf{H}_F$$

$$\mathbf{E}_{A(\theta,\phi)} = -j\omega \mathbf{A}_{(\theta,\phi)}, \qquad \mathbf{H}_{F(\theta,\phi)} = -j\omega \mathbf{F}_{(\theta,\phi)}$$

$$\mathbf{H}_A = \frac{1}{\eta}\hat{\mathbf{a}}_r \times \mathbf{E}_{A(\theta,\phi)}, \qquad \mathbf{E}_F = \eta \mathbf{H}_{F(\theta,\phi)} \times \hat{\mathbf{a}}_r$$

$$\mathbf{E} = -j\omega \mathbf{A}_{(\theta,\phi)} - j\omega\eta \mathbf{F}_{(\theta,\phi)} \times \hat{\mathbf{a}}_r$$

$$\mathbf{H} = -j\omega \mathbf{F}_{(\theta,\phi)} - j\omega \frac{1}{\eta}\hat{\mathbf{a}}_r \times \mathbf{A}_{(\theta,\phi)}$$

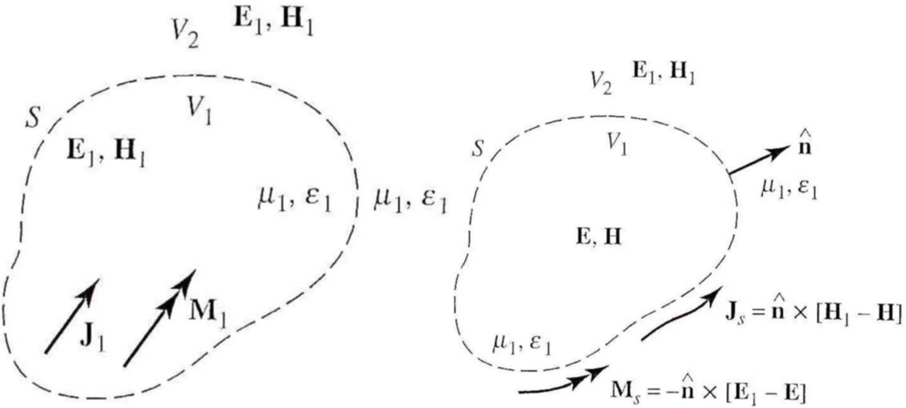


$$\mathbf{A} = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\iiint_V \mathbf{J}\left(\mathbf{r}'\right) e^{j\mathbf{k}\cdot\mathbf{r}'} dv', \qquad \mathbf{F} = \frac{\varepsilon}{4\pi}\frac{e^{-jkr}}{r}\iiint_V \mathbf{M}\left(\mathbf{r}'\right) e^{j\mathbf{k}\cdot\mathbf{r}'} dv'$$

$$\mathbf{k} = k\hat{\mathbf{a}}_r$$

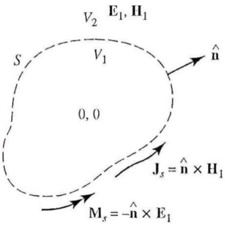
Outline

Field Equivalence Principle: Huygen's Principle

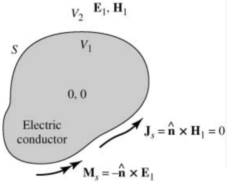


$$\mathbf{J}_s = \hat{\mathbf{n}} \times (\mathbf{H}_1 - \mathbf{H}), \qquad \mathbf{M}_s = -\hat{\mathbf{n}} \times (\mathbf{E}_1 - \mathbf{E})$$

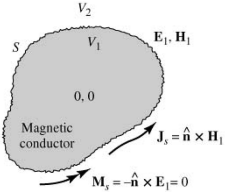
- Love's Equivalent



- Electric Conductor Equivalent



- Magnetic Conductor Equivalent



Outline

Image for Electric Conductor

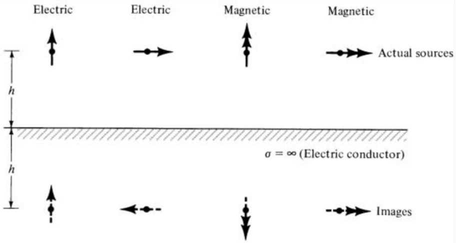
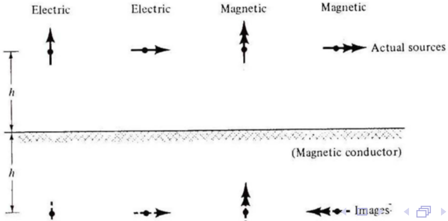
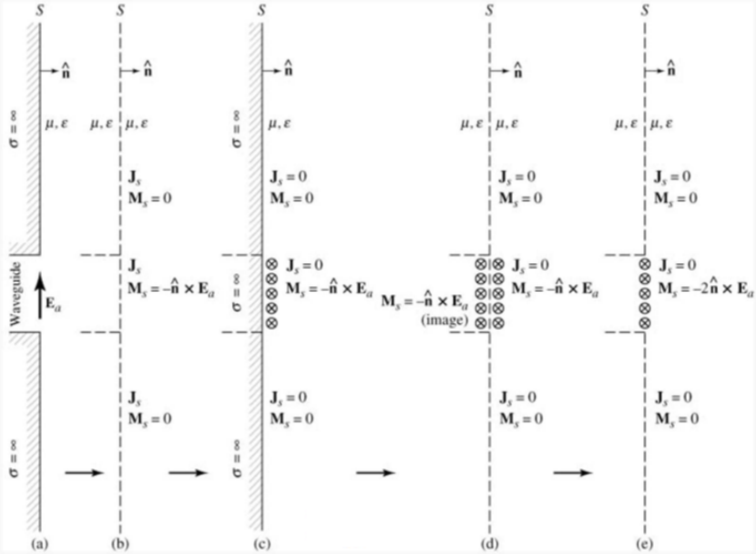


Image for Magnetic Conductor

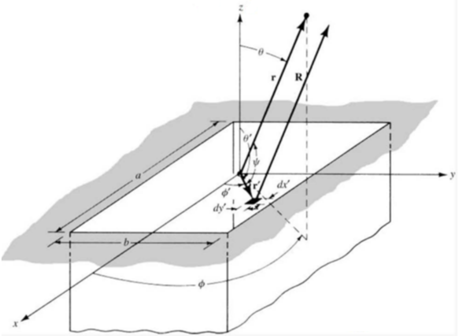


Outline

Equivalent Sources for Waveguide Aperture in an Infinite Ground Plane



Radiation from a Rectangular Aperture with Uniform Distribution in an Infinite Conductor

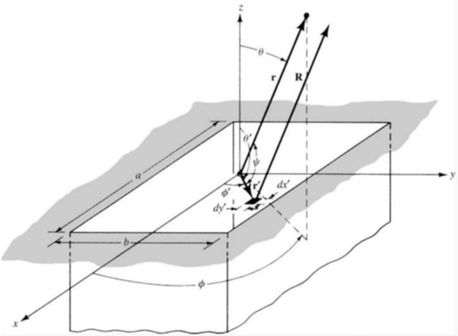


$$\mathbf{E} = \hat{\mathbf{a}}_y E_0, \qquad -a/2 \le x' \le a/2, \quad -b/2 \le y' \le b/2$$

Equivalent Currents:

$$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}, & -a/2 \le x' \le a/2, \quad -b/2 \le y' \le b/2 \\ \mathbf{0} & \text{elsewhere} \end{cases}, \qquad \mathbf{J}_s = 0$$

Radiation from a Rectangular Aperture with Uniform Distribution in an Infinite Conductor

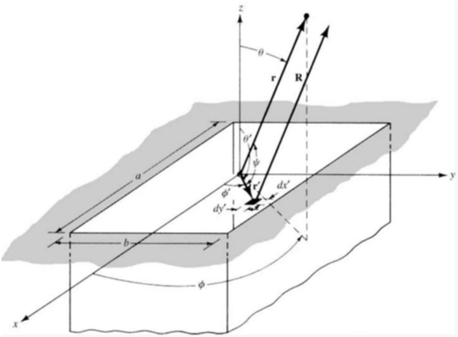


$$E_\theta = j\frac{abkE_0 e^{-jkr}}{2\pi r}\left[\sin\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$

$$E_\phi = j\frac{abkE_0 e^{-jkr}}{2\pi r}\left[\cos\theta\cos\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$

where $X = ka\sin\theta\cos\phi/2,\qquad Y = kb\sin\theta\sin\phi/2$

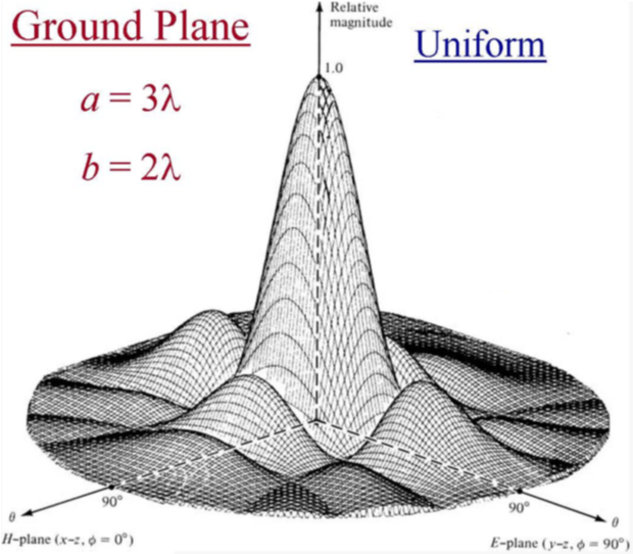
Radiation from a Rectangular Aperture with Uniform Distribution in an Infinite Conductor



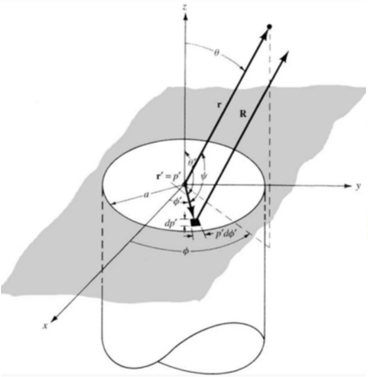
$$P_{\text{rad}} = \iint_{S_0} \mathbf{W}_{av} \cdot d\mathbf{s} = ab \frac{|E_0|^2}{2\eta}$$

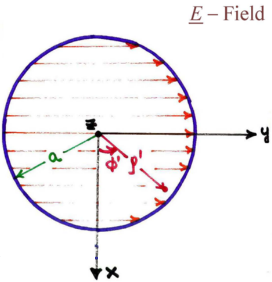
$$D = \frac{4\pi U}{P_{\text{rad}}}, \qquad D_0 = \frac{4\pi}{\lambda^2} A_p, \qquad \text{where } A_p = ab$$

Radiation from a Rectangular Aperture with Uniform Distribution in an Infinite Conductor



Circular Aperture in Ground Plane Illuminated with Uniform Field

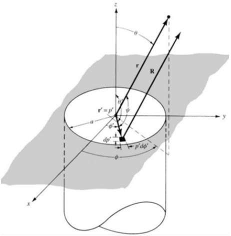




$$\mathbf{E}_a = E_0 \hat{\mathbf{a}}_y$$

$$\mathbf{M}_s = -2\hat{\mathbf{n}} \times \mathbf{E}_a = 2E_0 \hat{\mathbf{a}}_x$$

Circular Aperture in Ground Plane with Uniform Illumination



$$\mathbf{M}_s = -2\hat{\mathbf{n}} \times \mathbf{E}_a = 2E_0\hat{\mathbf{a}}_x$$

$$\cos\psi = \sin\theta\cos\left(\phi - \phi'\right)$$

$$\begin{bmatrix} M_r \\ M_\theta \\ M_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$H_\theta = \frac{-j\omega E_0 \varepsilon e^{-jkr}}{2\pi r}\cos\theta\cos\phi \iint e^{jk\rho'\sin\theta\cos(\phi-\phi')}\rho' d\phi' d\rho'$$

$$H_\phi = \frac{j\omega E_0 \varepsilon e^{-jkr}}{2\pi r}\sin\phi \iint e^{jk\rho'\sin\theta\cos(\phi-\phi')}\rho' d\phi' d\rho'$$

$$E_\theta = \eta H_\phi = \frac{jkE_0 e^{-jkr}}{2\pi r}\sin\phi \iint e^{jk\rho'\sin\theta\cos(\phi-\phi')}\rho' d\phi' d\rho'$$

$$E_\phi = -\eta H_\theta = \frac{jkE_0 e^{-jkr}}{2\pi r}\cos\theta\cos\phi \iint e^{jk\rho'\sin\theta\cos(\phi-\phi')}\rho' d\phi' d\rho'$$

Circular Aperture in Ground Plane with Uniform Illumination

Employing the two integral Identities:

$$\int_{2\pi} e^{jx\cos\phi'}\,d\phi' = 2\pi J_0(x)$$

$$\int xJ_0(\alpha x)\,dx = \frac{1}{\alpha}xJ_1(\alpha x) + C$$

$$E_\theta = \frac{jka^2E_0e^{-jkr}}{r}\sin\phi\,\frac{J_1(ka\sin\theta)}{ka\sin\theta}$$

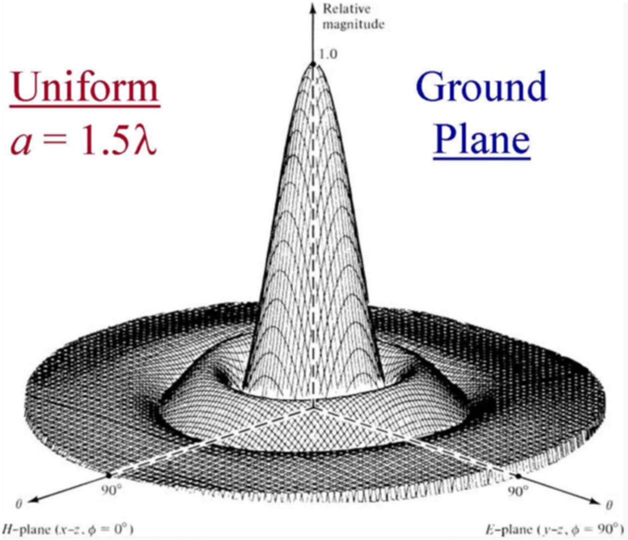
$$E_\phi = \frac{jka^2E_0e^{-jkr}}{r}\cos\theta\cos\phi\,\frac{J_1(ka\sin\theta)}{ka\sin\theta}$$

$$P_{\text{rad}} = \frac{|E_0|^2}{2\eta}\pi a^2, \qquad U = \frac{|E_0|^2}{2\eta}k^2a^4\left(\sin^2\phi + \cos^2\theta\cos^2\phi\right)\left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]^2$$

$$D = \frac{4\pi U}{P_{\text{rad}}} = 4k^2a^2\left(\sin^2\phi + \cos^2\theta\cos^2\phi\right)\left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]^2$$

$$\boxed{D_{\max}\big|_{\theta=0} = \frac{4\pi}{\lambda^2}\left(\pi a^2\right)}, \qquad \text{use } \lim_{x\to 0}\frac{J_1(x)}{x} = \frac{1}{2}$$

Circular Aperture in Ground Plane with Uniform Illumination



Outline

Aperture Effective Area

For Uniformly Illuminated Aperture,

$$D_{\max} = \frac{4\pi}{\lambda^2} A_p, \qquad \text{where } A_p \text{ is the aperture physical area.}$$

In general for no uniform illumination, maximum radiation is still at $\theta = 0$,

$$|E_{rad}|_{max} = \frac{k}{2\pi r} \left| \iint_{A_p} \mathbf{E}_a ds' \right|$$

$$U_{max} = \frac{1}{2\eta} \left(\frac{k}{2\pi} \left| \iint_{A_p} \mathbf{E}_a ds' \right| \right)^2, \qquad P_{\text{rad}} = \frac{1}{2\eta} \iint_{A_p} |\mathbf{E}_a|^2 \, ds'$$

$$D_{max} = \frac{4\pi U_{max}}{P_{\text{rad}}} = \frac{4\pi}{\lambda^2} A_{em}, \qquad \text{where } A_{em} \text{ is the maximum effective area.}$$

$$\boxed{A_{em} = \frac{\left| \iint_{A_p} \mathbf{E}_a ds' \right|^2}{\iint_{A_p} |\mathbf{E}_a|^2 \, ds'} \leq A_p}$$

Aperture Efficiency

Definition

Aperture Efficiency ε_{ap} is defines as,

$$\varepsilon_{ap} = \frac{A_{em}}{A_p},$$

where A_{em} is the *maximum effective area* and A_p is aperture *physical area*.

$$\varepsilon_{ap} = \frac{\left| \iint_{A_p} \mathbf{E}_a ds' \right|^2}{A_p \iint_{A_p} \left| \mathbf{E}_a \right|^2 ds'}$$